國立台灣科技大學九十五學年度碩士班招生試題

系所組別：光電工程研究所碩士班

科　　目：工程數學

總分 100 分

1. Solve $y'' + p(x)y' + q(x)y = f(x)$ by method of "variation of parameters"(10分)
You need to list your formulation step by step

2. Solve $y'' + 2ty' - 4y = 0 \quad y(0) = y'(0) = 0$ (10分)

3. Solve $(x^2+1)y'' + xy' - y = 0$ by power series method (10分)

4. A is a constant. Solve (10分)

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} + Ax \qquad (0 < x < L, t > 0)$$

$$y(0,t) = y(L,t) = 0 \qquad (t > 0)$$

$$y(x,0) = 0 \qquad (0 < x < L)$$

$$\frac{\partial y}{\partial t}(x,0) = 0 \qquad (0 < x < L)$$

5. Solve

(i) $\dfrac{dy}{dx} = \left(\dfrac{2x+y-1}{x-2}\right)^2$ (5分)

(ii) $\dfrac{dy}{dx} = \dfrac{2x+y-1}{4x+2y-4}$ (5分)

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6. Consider the matrix below :

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

(a). (4%) Find the determinant of **A**.

(b). (3%) Let **B** be the reduced row echelon form of **A**. Find **B**.

(c). (4%) Find the inverse matrix of **A**.

(d). (4%) Find the eigenvalues of **A**.

(e). (3%) Find an eigenvector of **A** corresponding to its second-largest eigenvalue.

7. Let $\mathcal{V}$ be the span of the set of vectors $S = \{(1,-1,3),(0,2,1),(1,3,5)\}$.

(a). (3%) What is the dimension of $\mathcal{V}$?

(b). (3%) Can we use S as a basis of $\mathcal{V}$?

8. Let $T : \mathcal{R}^2 \mapsto \mathcal{R}^2$ be a linear transformation. It is already known that $T((1,1)) = (2,-1)$, and $T((-1,1)) = (1,2)$.

(a). (4%) Find $T((x,y))$.

(b). (4%) Let Q be the inverse transformation of T. Find $Q((5,-3))$.

9. Consider the linear transformation $T : \mathcal{R}^2 \mapsto \mathcal{R}^2$, defined by $T((x,y)) = (2x+y, -x-2y)$.

(a). (4%) Let the two eigenvalues of T be denoted as λ_1 and λ_2, where $\lambda_1 \geq \lambda_2$. Find λ_1 and λ_2.

(b). (4%) Let $(a,1)$ be the eigenvector of T correspnding to λ_1, and let $(b,1)$ be the eigenvector of T correspnding to λ_2. Find a and b.

10. Let $\mathcal{V}$ denote the vector space of column vectors with two real elements. Let $\mathbf{v}_1 = [x_1 \;\; y_1]^T$ (superscript ^{T} means transpose) and $\mathbf{v}_2 = [x_2 \;\; y_2]^T$ be any vectors in $\mathcal{V}$. An inner product for $\mathcal{V}$ is defined by :

$$< \mathbf{v}_1 , \mathbf{v}_2 > = [x_1 \;\; y_1] \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}.$$

Consider the following vectors : $\mathbf{u} = [1 \;\; 2]^T$, and $\mathbf{w} = [-2 \;\; 1]^T$.

(a). (4%) Find the norms of **u** and **w**.

(b). (3%) Find the inner product of **u** and **w**.

(c). (3%) Is **u** orthogonal to **w** ?

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