國立台灣科技大學九十五學年度碩士班招生試題

系所組別：企業管理系碩士班甲組、乙組、丙組、甲一高職教師組、乙一高職教師組、丙一高職教師組

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「總分 100 分」

1. (25%)

(a) Suppose we have ten coins which are such that if the ith one is flipped then heads will appear with probability $i/10$, $i = 1, 2, \ldots, 10$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the fifth coin?

(b) Suppose that a random sample of 100 observations is taken from a normal distribution for which the mean θ is unknown and the variance is 1, and let $\tilde{N}_n$ denote the **sample median**.

Determine the value of $\Pr(|\tilde{N}_n-\theta| \leqq 0.1)$.

(c) Let Y be the random variable of the binomial distribution that have parameters $n = 100$ and $p = 0.5$. Approximate the value of $\Pr(Y = 50)$.

2. (25%)

(a) It is assumed that the random variables $X_1, \ldots, X_n$, form a random sample from a continuous distribution for which the d.f. F(x) is unknown, that θ denotes a median of this distribution. Also, let $Y_1 < Y_2 < \cdots < Y_n$ denote the order statistics of the sample. Thus, the random variable Y_1 is the smallest of the observations $X_1, \ldots, X_n$, the random variable Y_2 is the second smallest, and so on. Suppose n = 7 and the seven observed values in the sample are 7.11, 5.12, 8.44, 7.13, 7.12, 12.96, and 4.07. Determine the endpoints and the confidence coefficient for each of the following two confidence intervals for θ:

I_1: The interval between Y_2 and Y_4;

I_2: The interval between Y_3 and Y_5.

(b) Let two stochastically independent random variables Y_1 and Y_2, with binomial distributions that have parameters $n_1 = n_2 = 100$, p_1 and p_2, respectively, be observed to be equal to $y_1 = 55$ and $y_2 = 40$. Determine an approximate 90 percent confidence interval for $p_1 - p_2$.

(c) Suppose that 300 persons are selected at random from a large population and that each person in the sample is classified according to

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whether his blood type is O, A, B, or AB, and also according to whether his blood type is Rh positive or Rh negative. The observed numbers are given in the following Table. Test the hypothesis that the two classifications of blood types are independent.

	O	A	B	AB
Rh positive	82	89	54	19
Rh negative	13	27	7	9

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3. 假設每瓶「可口可樂」的瓶蓋內有英文字母 A, B, C, D 中的一個英文字母，而且不同英文字母的「可口可樂」在市場上各佔有 $1/4$ 的比例。今由市場中隨機買回 5 瓶「可口可樂」。若買回的 5 瓶中包含有 N 個不同英文字母。

   (a) (8%) 求算平均數 $\mathrm{E}(N)$。

   (b) (7%) 求算平均要買幾瓶才能集滿四個英文字母?

4. (14%) 假設 $\{X_1, \ldots, X_n\}$ 為一組抽自 $N(\mu, \sigma^2 = 2^2)$ 的「隨機樣本」，其中 μ 為未知。檢定 $H_0: \mu = 32$ 相對於 $H_1: \mu < 32$。「棄卻域」為

$$\left\{ (x_1, \ldots, x_n) \middle| \bar{x} = \sum_{i=1}^{n} x_i/n \leq c \right\}。$$

   若要求「型 I 錯誤」機率 $\alpha = 2.5\%$ 及在 $\mu = 30.8$ 時的「檢定力」為 84%，求算 n 及 c。
   (令 Z 具有標準常態分配，則 $\mathrm{P}(|Z| < 1) = 68\%$, $\mathrm{P}(|Z| < 2) = 95\%$。)

5. 試回答下列關於「單因子變異數分析」的問題。(必須針對問題簡要回答，否則不予計分)

   (a) (7%) 針對不同母體，為何需要相同變異數之假設 (於何時用上此假設)?

   (b) (7%) 直覺上，當「組間變異」偏大時，應傾向於棄卻或接受「虛無假設」? 為什麼?

6. (7%) 令 R^2 表示簡單線性迴歸 (模式: $Y_i = \alpha + \beta x_i + \epsilon_i$) 分析中之「判定係數」(coefficient of determination)。試針對大小不同 R^2 值 (即 $R^2 \approx 1$ 及 $R^2 \approx 0$) 畫出 (x, Y)之散布圖 (scatter-plot)。

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The t Distribution*

$$\Pr(T \le t) = \int_{-\infty}^{t} \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\,\Gamma(r/2)(1+w^2/r)^{(r+1)/2}}\,dw$$

$$[\Pr(T \le -t) = 1 - \Pr(T \le t)]$$

r	$\Pr(T \le t)$ = 0.90	0.95	0.975	0.99	0.995
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

The F Distribution*

$$\Pr(F \le f) = \int_0^f \frac{\Gamma[(r_1+r_2)/2](r_1/r_2)^{r_1/2}w^{r_1/2-1}}{\Gamma(r_1/2)\Gamma(r_2/2)(1+r_1w/r_2)^{(r_1+r_2)/2}}\,dw$$

$\Pr(F \le f)$	r_2	r_1 = 1	2	3	4	5	6	7	8	9	10	12	15
0.95	1	161	200	216	225	230	234	237	239	241	242	244	246
0.975		648	800	864	900	922	937	948	957	963	969	977	985
0.99		4052	4999	5403	5625	5764	5859	5928	5982	6023	6056	6106	6157
0.95	2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
0.975		38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4
0.99		98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
0.95	3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70
0.975		17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.4	14.3	14.3
0.99		34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9
0.95	4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86
0.975		12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66
0.99		21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2
0.95	5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62
0.975		10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43
0.99		16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72

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$\Pr(F \le f)$	r_2	r_1 = 1	2	3	4	5	6	7	8	9	10	12	15
0.95	6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94
0.975		8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27
0.99		13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56
0.95	7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51
0.975		8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57
0.99		12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31
0.95	8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22
0.975		7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10
0.99		11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52
0.95	9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01
0.975		7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77
0.99		10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96
0.95	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85
0.975		6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52
0.99		10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56
0.95	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62
0.975		6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18
0.99		9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01
0.95	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40
0.975		6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86
0.99		8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52

The Poisson Distribution

$$\Pr(X \le x) = \sum_{w=0}^{x} \frac{\mu^w e^{-\mu}}{w!}$$

x	$\mu = E(X)$ 0.5	1.0	1.5	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0	0.607	0.368	0.223	0.135	0.050	0.018	0.007	0.002	0.001	0.000	0.000	0.000
1	0.910	0.736	0.558	0.406	0.199	0.092	0.040	0.017	0.007	0.003	0.001	0.000
2	0.986	0.920	0.809	0.677	0.423	0.238	0.125	0.062	0.030	0.014	0.006	0.003
3	0.998	0.981	0.934	0.857	0.647	0.433	0.265	0.151	0.082	0.042	0.021	0.010
4	1.000	0.996	0.981	0.947	0.815	0.629	0.440	0.285	0.173	0.100	0.055	0.029
5		0.999	0.996	0.983	0.916	0.785	0.616	0.446	0.301	0.191	0.116	0.067
6		1.000	0.999	0.995	0.966	0.889	0.762	0.606	0.450	0.313	0.207	0.130
7			1.000	0.999	0.988	0.949	0.867	0.744	0.599	0.453	0.324	0.220
8				1.000	0.996	0.979	0.932	0.847	0.729	0.593	0.456	0.333
9					0.999	0.992	0.968	0.916	0.830	0.717	0.587	0.458
10					1.000	0.997	0.986	0.957	0.901	0.816	0.706	0.583
11						0.999	0.995	0.980	0.947	0.888	0.803	0.697
12						1.000	0.998	0.991	0.973	0.936	0.876	0.792
13							0.999	0.996	0.987	0.966	0.926	0.864
14							1.000	0.999	0.994	0.983	0.959	0.917
15								0.999	0.998	0.992	0.978	0.951
16								1.000	0.999	0.996	0.989	0.973
17									1.000	0.998	0.995	0.986
18										0.999	0.998	0.993
19										1.000	0.999	0.997
20											1.000	0.998
21												0.999
22												1.000

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The Chi-Square Distribution*

$$\Pr(X \le x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2}\, dw$$

r	Pr (X ≤ x) 0.01	0.025	0.050	0.95	0.975	0.99
1	0.000	0.001	0.004	3.84	5.02	6.63
2	0.020	0.051	0.103	5.99	7.38	9.21
3	0.115	0.216	0.352	7.81	9.35	11.3
4	0.297	0.484	0.711	9.49	11.1	13.3
5	0.554	0.831	1.15	11.1	12.8	15.1
6	0.872	1.24	1.64	12.6	14.4	16.8
7	1.24	1.69	2.17	14.1	16.0	18.5
8	1.65	2.18	2.73	15.5	17.5	20.1
9	2.09	2.70	3.33	16.9	19.0	21.7
10	2.56	3.25	3.94	18.3	20.5	23.2
11	3.05	3.82	4.57	19.7	21.9	24.7
12	3.57	4.40	5.23	21.0	23.3	26.2
13	4.11	5.01	5.89	22.4	24.7	27.7
14	4.66	5.63	6.57	23.7	26.1	29.1
15	5.23	6.26	7.26	25.0	27.5	30.6
16	5.81	6.91	7.96	26.3	28.8	32.0
17	6.41	7.56	8.67	27.6	30.2	33.4
18	7.01	8.23	9.39	28.9	31.5	34.8
19	7.63	8.91	10.1	30.1	32.9	36.2
20	8.26	9.59	10.9	31.4	34.2	37.6
21	8.90	10.3	11.6	32.7	35.5	38.9
22	9.54	11.0	12.3	33.9	36.8	40.3
23	10.2	11.7	13.1	35.2	38.1	41.6
24	10.9	12.4	13.8	36.4	39.4	43.0
25	11.5	13.1	14.6	37.7	40.6	44.3
26	12.2	13.8	15.4	38.9	41.9	45.6
27	12.9	14.6	16.2	40.1	43.2	47.0
28	13.6	15.3	16.9	41.3	44.5	48.3
29	14.3	16.0	17.7	42.6	45.7	49.6
30	15.0	16.8	18.5	43.8	47.0	50.9

The Normal Distribution

$$\Pr(X \le x) = N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[N(-x) = 1 - N(x)]$$

x	N(x)	x	N(x)	x	N(x)
0.00	0.500	1.10	0.864	2.05	0.980
0.05	0.520	1.15	0.875	2.10	0.982
0.10	0.540	1.20	0.885	2.15	0.984
0.15	0.560	1.25	0.894	2.20	0.986
0.20	0.579	1.282	0.900	2.25	0.988
0.25	0.599	1.30	0.903	2.30	0.989
0.30	0.618	1.35	0.911	2.326	0.990
0.35	0.637	1.40	0.919	2.35	0.991
0.40	0.655	1.45	0.926	2.40	0.992
0.45	0.674	1.50	0.933	2.45	0.993
0.50	0.691	1.55	0.939	2.50	0.994
0.55	0.709	1.60	0.945	2.55	0.995
0.60	0.726	1.645	0.950	2.576	0.995
0.65	0.742	1.65	0.951	2.60	0.995
0.70	0.758	1.70	0.955	2.65	0.996
0.75	0.773	1.75	0.960	2.70	0.997
0.80	0.788	1.80	0.964	2.75	0.997
0.85	0.802	1.85	0.968	2.80	0.997
0.90	0.816	1.90	0.971	2.85	0.998
0.95	0.829	1.95	0.974	2.90	0.998
1.00	0.841	1.960	0.975	2.95	0.998
1.05	0.853	2.00	0.977	3.00	0.999