

國立台灣科技大學九十五學年度碩士班招生試題

系所組別：資訊管理系碩士班甲組

科目：離散數學

(本試卷共七大題，滿分共計 100 分)

所有答案務必寫在答案卷上，寫於試題上不予計分

- 一、(10 points) Prove that there are no solutions in integers x and y to the equation

$$x^2 - 7y^3 = 3$$

- 二、A computer system considers a string of decimal digits a valid codeword if it contains an even number of 0 digits. For instance, 1230407869 is valid, whereas 120987045608 is not valid. Let a_n be the number of valid n -digit codewords.

(a) (8 points) Find a recurrence relation for a_n .

(b) (7 points) With the initial condition $a_1 = 9$, find an explicit formula for a_n .

- 三、Find a closed form for the generating function for the sequence $\{a_n\}$, where

(a) (7 points) $a_n = \frac{(-1)^{n+1}}{n}$ for $n = 1, 2, 3, \dots$

(b) (8 points) $a_n = n^2 + 3n + 2$ for $n = 0, 1, 2, 3, \dots$

- 四、Determine whether each of these sets is countable or uncountable, you just need to give the answers.

(a) (2 points) rational numbers

(b) (2 points) integers not divisible by 5

(c) (2 points) integers divisible by 3 but not by 7

(d) (2 points) the real numbers with decimal representations consisting of all 1s

(e) (2 points) the real numbers with decimal representations of all 1s or 2s



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- 五、The n -cube (or called hypercube) has 2^n vertices labeled $0, 1, \dots, 2^n-1$. An edge connects two vertices if the binary representation of their labels differs in exactly one bit. Answer the following questions.
- (a) (5 points) What is the number of edges for the n -cube?
 - (b) (5 points) Does the n -cube contain a Hamiltonian cycle? If your answer is "yes", prove it. Otherwise, give a counterexample.
 - (c) (5 points) Does the n -cube contain an Euler cycle? If your answer is "yes", prove it. Otherwise, give a counterexample.
 - (d) (5 points) For a coloring of a graph, any vertex has a color different from that of any adjacent vertex. What is the smallest number of colors for the n -cube?

- 六、There are two routes from city A to city D . One route passes through city B and the other route passes through city C . During the period 8:00 A.M. to 9:15 A.M., the average trip times are

A to B	30 minutes
A to C	15 minutes
B to D	15 minutes
C to D	15 minutes

The maximum capacities of the routes are

A to B	1000 vehicles
A to C	3000 vehicles
B to D	4000 vehicles
C to D	2000 vehicles

- (a) (5 points) Represent the flow of traffic from A to D during the period 8:00 A.M. to 9:15 A.M. as a network.
- (b) (5 points) Find the maximal flow in the network got from (a).
- (c) (5 points) Find the minimum cut in the network got from (a).

- 七、Given the grammar $T=\{a, b\}$, $N=\{\sigma, A\}$, and starting symbol σ , with productions

$$\begin{aligned} \sigma &\rightarrow b\sigma, & \sigma &\rightarrow aA, & A &\rightarrow a\sigma, \\ A &\rightarrow bA, & A &\rightarrow a, & \sigma &\rightarrow b. \end{aligned}$$

- (a) (5 points) Write out the corresponding regular expression.
- (b) (5 points) Find a non-deterministic finite-state automaton that accepts this grammar.
- (c) (5 points) Find a deterministic finite-state automaton that accepts this grammar.

