

國立台灣科技大學九十六學年度碩士班招生試題

系所組別： 自動化及控制研究所碩士班甲組、乙組

科 目： 工程數學

總分為100分，題號請標示清楚。

1. Solve the following differential equations:

(1) $xy' - 2y + y^2 = 0$ (10%)

(2) $x^2y'' - 2xy' + 2y = \ln(x) - 1$ (10%)

2. Use the Laplace Transform method to solve the following system with initial conditions. (20%)

$$x' - 2y' + 3z = 0$$

$$x - 4y' + 3z' = t$$

$$x - 2y' + 3z' = -1$$

$$x(0) = 0, y(0) = 0, z(0) = 0$$

3. Find the first five nonzero terms of the **Maclaurin series** solution of the initial value problem. (10%)

$$y'' + \frac{1}{x+2}y' - xy = 0; y(0) = y'(0) = 1$$

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4. Determine the Fourier series for the function

$$f(x) = 3x + 8x^2, \quad -2 \leq x \leq 2 \quad (10\%).$$

5.

(a) Find singular points of the function $\frac{1}{\sin\left(\frac{\pi}{z}\right)}$. (10%)

(b) Evaluate the integral

$$\int_C \frac{5z-2}{z(z-1)} dz,$$

where C is the circle $|z| = 2$, described counterclockwise. (10%)

6.

(a) Consider an irrotational vector field

$\mathbf{u} = (x + 3y + az)\mathbf{i} + (bx - 5y - 2z)\mathbf{j} + (4x + cy + z)\mathbf{k}$. Please determine coefficients a, b , and c . Also, determine a potential function ϕ which satisfies $\nabla\phi = \mathbf{u}$. (10%)

(b) Determine the following surface integral by the divergence theorem

$$\iiint_S (x+z)dydz + (y+z)dzdx + (x+y)dxdy,$$

where $S: x^2 + y^2 + z^2 = 9$. (10%)