國立台灣科技大學九十六學年度碩士班招生試題

系所組別： 機械工程系碩士班乙組、丙組、丁組

科　　目： 工程數學

總分 **100** 分

1. Find the general solution of differential equation

$$x^2y'' + 5xy' - 12y = 12\ln x \qquad (15\%)$$

2. Find the solution of equation

$$y' + 3y + 2\int_0^t y\,dt = f(t) \quad ; y(0) = 1, \text{ with}$$

$$f(t) = \begin{cases} 0 & for\ 0 \le t < 1 \\ 1 & for\ t \ge 1 \end{cases} \qquad (15\%)$$

3. Suppose that AX = C is a linear system of order 3, and that to the C vector

$C = \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$, there correspond unique solutions $X = \begin{pmatrix}2\\5\\-1\end{pmatrix}, \begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}3\\0\\4\end{pmatrix}$,

respectively. Find

(a) the matrix A, and

(b) the solution X corresponding to $C = \begin{pmatrix}4\\3\\-1\end{pmatrix}$. (20%)

4. Expand the function in a Fourier integral and determine what the integral representation converges to.

$$f(x) = \begin{cases} \sin x, & -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2} \\ 0, & |x| > \dfrac{\pi}{2} \end{cases} \qquad (15\%)$$

5. Apply the residue theorem to evaluate

$$\int_0^{2\pi} \frac{d\theta}{a + b(\cos\theta)} \qquad \text{where} \quad a < b < 0 \qquad (15\%)$$

6. Solve the following boundary value problem

$$\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial y^2} = 0 \qquad (0 < x < 1,\ \ 0 < y < 1)$$

$$u(0, y) = u(1, y) = 0, \quad u(x,0) = 0, \text{ and } \quad u(x,1) = 50 \qquad (20\%)$$