

## 國立台灣科技大學九十六學年度碩士班招生試題

系所組別：化學工程系碩士班

科目：工程數學

總分 100 分，請依序作答，並詳列計算過程。

1. Find the general solution of the system by diagonalizing the coefficient matrix.

$$\frac{dX_1}{dt} = 3X_1 - X_2 + X_3,$$

$$\frac{dX_2}{dt} = X_1 + X_2 - X_3,$$

$$\frac{dX_3}{dt} = X_1 - X_2 + X_3 \quad (15\%)$$

2. Solve the boundary value problem using separation of variables.

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < L, t > 0$$

$$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(L,t)}{\partial x} = 0 \quad \text{for } t > 0$$

$$u(x,0) = \begin{cases} A & \text{for } 0 \leq x \leq \frac{L}{2} \\ 0 & \text{for } \frac{L}{2} < x \leq L \end{cases} \quad (20\%)$$

3. Find the fourier series of
- $f(x)$
- .

$$f(x) = \begin{cases} 0 & \text{for } -3 \leq x \leq 0 \\ x & \text{for } 0 \leq x \leq 3 \end{cases} \quad (10\%)$$

4. Determine all values of
- $Z^n$
- .

$$(1-i)^{1/3} \quad (5\%)$$

5. Solve the given differential equation.

$$y' + y = f(t); \quad y(0) = 4, \quad \text{where } f(t) = \begin{cases} 0 & \text{for } 0 \leq t < \pi \\ 2 \cos(t) & \text{for } t \geq \pi \end{cases} \quad (15\%)$$

6. Solve the given differential equation.

$$y'' + y(y')^3 = 0 \quad (10\%)$$

7. Find the complete general solution of the given differential equation.

$$(1-2x-x^2)y'' + 2(1+x)y' - 2y = 0; \quad y_1 = x+1 \quad (15\%)$$

8. Evaluate
- $\iint_S (2xy/z) dS$
- , where
- $S$
- is the part of a paraboloid surface
- $z = x^2 + y^2$
- lying in the first octant between the
- $xy$
- plane and the
- $z = 2$
- planes. (10%)