

國立台灣科技大學九十六學年度碩士班招生試題

系所組別：電子工程系碩士班乙一組

科目：工程數學

總分 100 分

1. (15 Points) Briefly answer the following questions. You will not get any credit if only the answer is given.

- (a) (5 Points) Let $C = AB$, where A , B , and C are $m \times n$, $n \times m$, and $m \times m$ matrices, respectively ($m > n$). Is C invertible? Briefly justify your answer.
- (b) (5 Points) Consider the subspace V of \mathcal{R}^4 defined as

$$V = \{(x_1, x_2, x_3, x_4) \mid x_3 = -3x_4, x_1 + 2x_2 = 0\}$$

Determine $\dim(V)$.

- (c) (5 Points) Let D be a 4×4 matrix with real entries. Suppose that the diagonal elements of D are all equal to 2, i.e. $d_{11} = d_{22} = d_{33} = d_{44} = 2$ and that D is singular. If we know one of its eigenvalue is $2 + i$, then determine the other three eigenvalues.
2. (10 Points) Let matrix A be de given by

$$A = \begin{bmatrix} 3 & 6 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

- (a) (5 Points) Determine elementary matrices, E_1 , E_2 , E_3 such that $E_1 E_2 E_3 A = L$, where L is a lower triangular matrix.
- (b) (5 Points) From (a); factorize A as $A = UL$, where U is an upper triangular matrix and L is as given in (a).
3. (8 Points) Consider a linear transformation T

$$T : \mathcal{R}^3 \rightarrow \mathcal{R}^4$$

with

$$T\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = \begin{bmatrix} 2a_1 \\ a_1 + 2a_2 \\ a_2 + 2a_3 \\ a_3 + 2a_1 \end{bmatrix}$$

Is T one-to-one? Does T map \mathcal{R}^3 onto \mathcal{R}^4 ? Justify your answers.

4. (7 Points) Let \mathbf{u} be a unit column vector in \mathcal{R}^3 that is perpendicular to the plane U which passes through the origin. Given a vector \mathbf{x} as shown in the following figure. PLOT the vector $\mathbf{y} = A\mathbf{x}$, where

$$A = I + \mathbf{u}\mathbf{u}^T$$

where I is a 3×3 identity matrix.



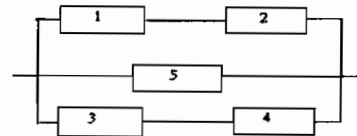
5. (10 Points) Consider a bottle which contains two types of viruses, type β and type η . Assume that each day 30% of the type β viruses will turn to type η , while 40% of the type η viruses will turn to type β . Suppose that the total number of viruses in the bottle remains a constant. Determine the percentage of type β viruses in the bottle after a long run. You can assume any initial condition.

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6. 如圖下所示的電路圖，假設每個元件正常工作的機率為 P ，且所有元件都獨立地工作，求該系統能正常工作的機率？(6 分)



7. 設甲、乙、丙三飛彈向同一敵機射擊，甲、乙、丙擊中敵機的機率分別為 0.4、0.5、0.7，如只有一個飛彈擊中，飛機墜毀機率為 0.2；如果有兩飛彈擊中，飛機墜毀的機率為 0.6，如果有三個飛彈擊中，飛機墜毀機率為 0.9，如果已知飛機墜毀，求兩飛彈擊中的機率為多少？(11 分)

8. 如已使用了 t 小時的電子零件在以後 Δt 小時內損毀壞的機率為 $\lambda \Delta t + o(\Delta t)$ ，其中 λ 為正常數，求電子零件在 T 小時內損毀的機率？(11 分)

9. 設隨機變量 (X, Y) 的聯合密度函數為

$$p(x, y) = \begin{cases} 1, & |y| < x, 0 < x < 1 \\ 0, & \text{其他} \end{cases}$$

求 $p(x|y)$ ， $p(y|x)$ ， $p\left\{X > \frac{1}{2} \mid Y > 0\right\}$ ， $p\left\{Y > \frac{1}{2} \mid X > \frac{1}{2}\right\}$? (11 分)

10. 一信號 $x(t) = A \cos(2\pi f_c t + \Theta)$ 其中 A 、 f_c 為常數， Θ 為一 uniform distribution 之 Random variable over 0 到 2π ，求此信號的 autocorrelation function？(11 分)