

國立台灣科技大學九十六學年度碩士班招生試題

系所組別：電子工程系碩士班乙二組

科目：工程數學

總分/100分

請標題號,並依題號順序作答

1. Solve the following equations.

(a). $(3x^2 \cos 4y - 2xy)dx - (4x^3 \sin 4y + x^2)dy = 0$ (5%)

(b). $y^{(iv)} - 2y'' = 0$ (5%)

2. Solve the following problem.

(a). $y'' + 4y' + 3y = 3\delta(t-2) + H(t-1)$; $y(0) = y'(0) = 0$ ($H(t)$ is the unit step and $\delta(t)$ is the unit impulse function.) (5%)

(b). $f(t) = 3t^5 + \int f(t-\tau)e^{-\tau} d\tau$ (10%)

3. Solve $e^{x+y} y^{-2} x^2 + e^{x+2y} xy' = 0$ (10%)

4. Solve the following system of differential equations.

$$\begin{cases} (D^2 + 1)x + (D - 1)y = 4e^t + 1 \\ (D + 1)x + (D - 1)y = 2 \end{cases}; x'(0) = x(0) = y'(0) = y(0) = 0 \quad (15\%)$$

國立台灣科技大學九十六學年度碩士班招生試題

系所組別：電子工程系碩士班乙二組

科目：工程數學

第二頁

5. (a) A is a 4×4 matrix satisfying $\det(3I + A) = 0$, $AA^T = 2I$, and $\det A < 0$. Find one eigenvalue of adjoint matrix of A . (10%) (b) B is a matrix of $n \times n$ satisfying $\text{rank}(B + I) + \text{rank}(B - I) = n$. Find B^2 . (10%)
6. (a) A is an $n \times m$ matrix and B is a $m \times n$ matrix, where $n < m$. I is an $n \times n$ identity matrix. Suppose $AB = I$, are the row vectors of B linearly independent or dependent? Prove your answer. (5%) (b) Matrix C has eigenvectors and the corresponding eigenvalues \bar{X}_1 , \bar{X}_2 and λ_1 , λ_2 , respectively, where $\lambda_1 \neq \lambda_2$. Is $\bar{X}_1 + \bar{X}_2$ also an eigenvector of C ? Prove your answer. (5%)
7. If A is a matrix of 4×4 . Let $\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4$ be the row vectors of A , where $\bar{a}_2, \bar{a}_3, \bar{a}_4$ are linearly independent and $\bar{a}_1 = 2\bar{a}_2 - \bar{a}_3$. If $\bar{Y} = \bar{a}_1 + \bar{a}_2 + \bar{a}_3 + \bar{a}_4$, find the general solution of $A\bar{X} = \bar{Y}$. (10%)
8. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and $AM = BA$, where M is a 3×3 matrix. Find M^5 . (10%)