

國立台灣科技大學九十六學年度碩士班招生試題

系所組別：電子工程系碩士班乙三組

科目：工程數學

總分 100 分

1. (8 points) Briefly answer the following questions. You will not get any credit if only the answer is given.

- (a) (4 points) Let $C = AB$, where A , B , and C are $m \times n$, $n \times m$, and $m \times m$ matrices, respectively ($m > n$). Is C invertible? Briefly justify your answer.
- (b) (4 points) Let D be a 4×4 matrix with real entries. Suppose that the diagonal elements of D are all equal to 2, i.e. $d_{11} = d_{22} = d_{33} = d_{44} = 2$ and that D is singular. If we know one of its eigenvalue is $2 + i$, then determine the other three eigenvalues.

2. (8 points) Let matrix A be de given by

$$A = \begin{bmatrix} 3 & 6 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

- (a) (4 points) Determine elementary matrices, E_1 , E_2 , E_3 such that $E_1 E_2 E_3 A = L$, where L is a *lower triangular* matrix.
- (b) (4 points) From (a), factorize A as $A = UL$, where U is an *upper triangular* matrix and L is as given in (a).

3. (8 points) Consider a linear transformation T

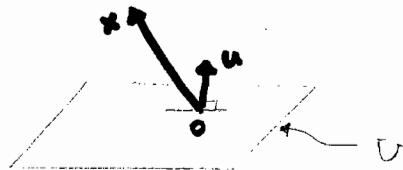
$$T : \mathcal{R}^3 \rightarrow \mathcal{R}^4$$

with

$$T\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = \begin{bmatrix} 2a_1 \\ a_1 + 2a_2 \\ a_2 + 2a_3 \\ a_3 + 2a_1 \end{bmatrix}$$

Is T one-to-one? Does T map \mathcal{R}^3 onto \mathcal{R}^4 ? Justify your answers.

4. (11 points) Let u be a unit column vector in \mathcal{R}^3 that is perpendicular to the plane U which passes through the origin. Given a vector x as shown in the following figure.



Suppose that

$$A = I + uu^T$$

where I is a 3×3 identity matrix.

- (a) (7 points) PLOT the vector y , where $y = Ax$.
- (b) (4 points) Is u an eigenvector of A ? If no, explain why. If yes, determine the corresponding eigenvalue.
5. (15 points) Consider a differential equation of the form

$$y''(t) + y'(t) - 2y(t) = e^t$$

with the initial condition $y(0) = 2$ and $y'(0) = 1$. Please find an explicit solution of this differential equation.

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6. (15 Points) A partial differential equation is defined as

$$\frac{\partial u(x, y)}{\partial x} = 2 \frac{\partial u(x, y)}{\partial y} + 2u(x, y).$$

The boundary conditions of this partial differential equation is given by

$$u(x, 0) = e^x + 2e^{-4x}.$$

Please find the solution of this partial differential equation.

7. (15 Points) It is a well-known fact that a complex-variable function $f(z)$ with well-defined derivative at a point $z = z_0$ may not be analytic at $z = z_0$. Please give such an example and verify the above property for this complex-variable function.
8. (10 Points) Suppose n is a positive integer and z_0 is a complex constant. Please determine the residue of the function $e^{2z}/(z - z_0)^n$ at the pole $z = z_0$.
9. (10 Points) Please derive an explicit formula for computing $\sin^{-1}(z)$, where z is a complex number.