國立台灣科技大學九十六學年度碩士班招生試題

系所組別： 電子工程系碩士班丙組

科　　目： 工程數學

總分100分

請標題號,並依題號順序做答

1. Solve the following equations.

(a). $(3x^2\cos 4y-2xy)dx - (4x^3\sin 4y+x^2)dy=0$ (5%)

(b). $y^{(iv)} - 2y''=0$ (5%)

2. Solve the following problem.

(a). $y''+4y'+3y=3\delta(t-2)+H(t-1)$; $y(0)=y'(0)=0$ (H(t) is the unit step and δ(t) is the unit impulse function.) (5%)

(b). $f(t)=3t^5+\int_0^t f(t-\tau)e^{-\tau}d\tau$ (10%)

3. Solve $e^{x+y}y^{-2}x^2+e^{x+2y}xy'=0$ (10%)

4. Solve the following problem.

$$\frac{\partial^2 y}{\partial t^2}=4\frac{\partial^2 y}{\partial x^2} \quad for \quad 0<x<3, t>0$$

$$y(0,t)=y(3,t)=0 \quad for \quad t\ge 0$$

$$y(x,0)=0, \frac{\partial y}{\partial t}(x,0)=g(x) \quad for \quad 0\le x<3,$$

where

$$g(x)=\begin{cases} 0 & for \quad 0\le x<2 \\ 3-x & for \quad 2\le x<3 \end{cases} \qquad (15\%)$$

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5. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & 1 & 0 \end{bmatrix}.$$

(a). (4%) Let $\det(\mathbf{A})$ denote the determinant of $\mathbf{A}$. Then, $\det(\mathbf{A}) =$?

(b). (5%) Find $\mathbf{A}^{-1}$ (i.e. the inverse matrix of $\mathbf{A}$).

6. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}.$$

(a). (4%) Find the eigenvalues of $\mathbf{A}$.

(b). (4%) Find a matrix $\mathbf{P}$ so that $\mathbf{P}^{-1}\mathbf{A}\,\mathbf{P} = \mathbf{D}$, where $\mathbf{D}$ is a diagonal matrix. Then, write down the matrix $\mathbf{D}$.

(c). (4%) $\mathbf{A}^{100} + \mathbf{A}^{199} =$?
< Hint > Use an implied result from (b) : $\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.

7. Let $T : \mathcal{R}^2 \mapsto \mathcal{R}^3$ be a linear transformation. Assume that $T(1,2) = (1,0,-1)$, and $T(0,-1) = (2,1,0)$.

(a). (4%) If $T(x,y) = (ax+by, cx+dy, ex+fy)$, then $(a+b+c+d+e+f) =$?

(b). (4%) What is the dimension of the kernel (also called null space) of T?

(c). (4%) Let us adopt the ordered bases $B = \{(1,0),(0,1)\}$ for $\mathcal{R}^2$, and $D = \{(1,0,0),(0,1,0),(0,0,1)\}$ for $\mathcal{R}^3$. Find the matrix for T with respect to the bases B and D.

8. Let us consider the inner product space $\mathcal{V} = \{(x_1,x_2,x_3,x_4)|x_1,x_2,x_3,x_4 \in \mathcal{R}\}$, where $\mathcal{R}$ denotes the set of real numbers. The vector additions and scalar multiplications are the usual ones. In other words,

$$(x_1,x_2,x_3,x_4) + (y_1,y_2,y_3,y_4) = (x_1+y_1, x_2+y_2, x_3+y_3, x_4+y_4)\ ,$$

and

$$c \cdot (x_1,x_2,x_3,x_4) = (cx_1, cx_2, cx_3, cx_4)\ .$$

The inner product is defined according to

$$< (x_1,x_2,x_3,x_4),(y_1,y_2,y_3,y_4) > = x_1y_1 + 2x_2y_2 + 2x_3y_3 + 3x_4y_4\ .$$

Let us consider the four vectors in $\mathcal{V}$: $\mathbf{v}_1 = (0,1,-1,0)$, $\mathbf{v}_2 = (2,1,0,1)$, $\mathbf{v}_3 = (1,0,0,0)$, and $\mathbf{v}_4 = (3,3,-2,1)$.

(a). (4%) Are $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ linearly independent?

(b). (4%) Let S be the span of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$. Then, what is the dimension of S?

(c). (4%) Find the norm of $\mathbf{v}_1$.

(d). (5%) Let us denote the norm of $\mathbf{v}_1$ by $||\mathbf{v}_1||$ and then define $\mathbf{u}_1 = (1/||\mathbf{v}_1||) \cdot \mathbf{v}_1$. Find a vector $\mathbf{u}_2$ so that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthonormal basis for the span of $\mathbf{v}_1$ and $\mathbf{v}_2$.