國立台灣科技大學九十六學年度碩士班招生試題

系所組別：電機工程系碩士班乙一組、乙二組、丙三組、乙二高職教師組

科　　目：微分方程及線性代數

總分100分

1. (10 points) Solve the second order differential equation for **y(x)**:

$$x^2 y'' - 11xy' + 27y = 2\ln x + 1, \quad x > 0$$

2. (10 points) Solve the system of differential equations and initial conditions for both **x(t)** and **y(t)**:

$$\begin{cases} 2\dfrac{dx}{dt} + \dfrac{dy}{dt} + 4x - y = \delta(t) \\ \dfrac{dx}{dt} + 2\dfrac{dy}{dt} + 2x = 1 \\ x(0) = y(0) = 0 \end{cases}$$

3. (15 points) Solve the partial differential equation for **y(x,t)**:

$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} = 0 \qquad \text{for} \quad 0 < x < a, t > 0$$

$$\frac{\partial}{\partial x} y(0,t) = \frac{\partial}{\partial x} y(a,t) = 0 \qquad \text{for} \quad t \ge 0$$

$$y(x,0) = 0 \qquad \text{for} \quad 0 \le x \le a$$

$$\frac{\partial}{\partial t} y(x,0) = f(x) \qquad \text{for} \quad 0 \le x \le a$$

$$f(x) = \frac{1}{27}\cos\frac{4\pi x}{a}$$

4. (15 points) Use Laplace transform to solve the differential equation

$$y'' + y = f(t) \quad \text{where} \quad f(t) = \begin{cases} 0 & 0 \le t < \pi \\ 1 & \pi \le t < 2\pi \\ 0 & t \ge 2\pi \end{cases} \quad \text{and} \quad y(0) = 0, \quad y'(0) = 1$$

5. (a) (4 points) Let $y_1 = (1,1,0)$, $y_2 = (2,0,1)$, $y_3 = (2,2,1)$, prove that the set $\{y_1, y_2, y_3\}$ is linearly independent.

(b) (8 points) Use the Gram-Schmidt process to find an orthogonal basis $\{x_1, x_2, x_3\}$ from $\{y_1, y_2, y_3\}$ with $x_1 = y_1$.

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6. (8 points) Use Gaussian elimination to solve the following system of linear equations

$$\begin{aligned} 3x_1 - x_2 + 2x_3 + 4x_4 + x_5 &= 2 \\ x_1 - x_2 + 2x_3 + 3x_4 + x_5 &= -1 \\ 2x_1 - 3x_2 + 6x_3 + 9x_4 + 4x_5 &= -5 \\ 7x_1 - 2x_2 + 4x_3 + 8x_4 + x_5 &= 6 \end{aligned}$$

7. (10 points; 5 points each) Prove the following theorem:

If L: V → W is a linear transformation and S is a subset of V, then

(i) *the kernel of L,* ker(*L*) *is a subset of V.*

(ii) *L(S) is a subspace of W.*

8. (20 points)Prove the following theorem:

An n x n matrix A is diagnosable if and only if A has n linearly independent eigenvectors.