國立台灣科技大學九十六學年度碩士班招生試題

系所組別：電機工程系碩士班丙一組、丙二組

科 目：線性代數與機率

總分100分

1. (10%) Let $\mathbf{u}_1 = (2,1)^T$, $\mathbf{u}_2 = (5,3)^T$ and let L be the linear operator that rotates vectors in R^2 by $45°$ in the counterclockwise direction. Find the matrix representation of L with respect to the ordered basis $[\mathbf{u}_1, \mathbf{u}_2]$.

2. (15%) Use the fact that the matrices A and U are row equivalent, where

$$A = \begin{pmatrix} 1 & 2 & w & 0 & 0 \\ 2 & 5 & x & 1 & 0 \\ 3 & 7 & y & 2 & -2 \\ 4 & 9 & z & -1 & 3 \end{pmatrix} \text{ and } U = \begin{pmatrix} 1 & 0 & 3 & 0 & -3 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) (4%) What are the values of w, x, y and z?
(b) (5%) Find a basis for the nullspace of A.
(c) (3%) Find the dimension of the row space of A^T.
(d) (3%) How many solutions will the linear system $A^T x = b$ have if b is in the column space of A^T?

3. (15%) Let $A = \begin{pmatrix} 4 & 2 & 2 \\ -5 & -3 & -2 \\ 5 & 5 & 4 \end{pmatrix}$

(a) (3%) Find the eigenvalues of A.
(b) (3%) Is A diagonalizable? Explain.
(c) (5%) Show that A and B have the same eigenvalues if B is similar to A.
(d) (4%) Find the determinant of $-A^3$.

4. (10%) Find the least squares linear approximation, $g(x) = a + bx$, to $f(x) = x^3$ on the interval $[0, 2]$ by forming an orthonormal basis first.

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5. (15%)

(1) Find the probability of $A \cup B \cup C$, where A, B and C are independent and $P(A) = 0.3$, $P(B) = 0.5$ and $P(C) = 0.2$.

(2) Explain the relationship of uncorrelatedness and independence.

(3) Let X and Y be zero-mean, unit-variance independent Gaussian random variables. Find the value of R for which the probability (X,Y) falls inside a circle of radius R is 1/2.

6. (15%)

(1) Let X be the number of customers waiting for a bus, where the probability that k customers are waiting is $P(X=k) = p(1-p)^{k-1}$, $k = 1,2....$ Suppose that the bus can take M passengers. Find the probability mass function for the number of customers left behind.

(2) Let X and Y be independent random variables that are uniformly distributed in the interval [-1,1]. Find the probability density function of $Z = XY$.

7. (20%) Let the number of jobs arriving at a shop in a 1-hour period be a Poisson random variable N with mean $m = 4$, i.e., $P(N=k) = \frac{m^k}{k!}e^{-m}$, $k = 0,1,...$ Each job requires X_j seconds to complete, where X_j's are independent and identically distributed random variables that are equal to 3 minutes or 6 minutes with equal probability.

(1) Find the mean and variance of the total work W (measured in minutes) arriving in a 1-hour period.

(2) Find the probability generating function $G_W(z) = E[z^W]$.