

國立台灣科技大學九十六學年度碩士班招生試題

系所組別：資訊工程系碩士班

科 目：計算機數學

總分 100 分

- Assume that births are independent and the probability of a birth in each month is $\frac{1}{12}$.
 - (5%) What is the probability that in a group of n people, there are at least two born in the same month of the year?
 - (5%) How many people are needed to make the probability greater than $\frac{1}{2}$ that there are at least two people born in the same month of the year?
- (5%) What is Chinese Remainder Theorem? State it in mathematical expression.
 - (8%) Find a simultaneous solution for the system of four congruences as follows:

$$\begin{aligned}x &\equiv 1 \pmod{2} \\x &\equiv 2 \pmod{3} \\x &\equiv 3 \pmod{5} \\x &\equiv 5 \pmod{7}\end{aligned}$$

- (12%) Solve the following system of recurrence relations.

$$\begin{aligned}a_{n+1} &= -2a_n - 4b_n \\b_{n+1} &= 4a_n + 6b_n \\n &\geq 0, a_0 = 1, b_0 = 0\end{aligned}$$

- (5%) Suppose that a connected planar simple graph has 18 vertices, each of degree 4. How many regions does a representation of this planar graph split the plane?

Figure 1:

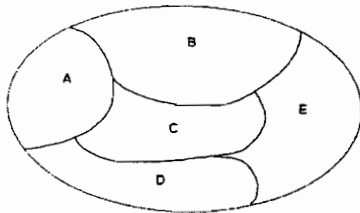
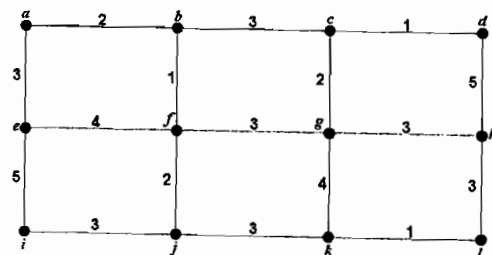


Figure 2:



- (5%) Construct the *dual graph* for the planar map shown in Figure 1 and find its *chromatic number*.
- (5%) Plot a spanning tree with *minimal total weight* containing the edge $\{e, i\}$ in the weighted graph shown in Figure 2.

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7. True or False? No point will be given without correct justification.
- (a) (5%) Let $A \in R^{n \times n}$ and $b \in R^n$. If $A^T = A$ and $Ax = Az = b$ then $x = z$.
 - (b) (5%) Let $V = (M, +, \cdot)$ be a vector space, where M is the set of $n \times n$ matrices. Then the set consisting of all *nonsingular* matrices is a subspace of V .
 - (c) (5%) The linear system $(A^T A + I)x = b$ always has a unique solution where $A \in R^{m \times n}$ and $I \in R^{n \times n}$ is an identity matrix.
8. (a) (5%) What is *LU*-decomposition?
- (b) (8%) Find a *QR*-decomposition of $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.
9. Let A be a 3×4 matrix and its nullspace has a basis $(2 \ 3 \ 1 \ 0)^T$.
- (a) (4%) What is the rank of A and the complete solution of $Ax = 0$?
 - (b) (8%) What is the reduced row echelon form of A ?
10. (10%) Let $T : P_2 \rightarrow M_{22}$ be a linear transformation such that $T(a + bx + cx^2) = \begin{pmatrix} a & b+c \\ a+b & c \end{pmatrix}$, find the matrix representation of T with respect to the standard bases. **Note:** The standard basis of $P_2 = \{1, x, x^2\}$ and the standard basis of

$$M_{22} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$