國立台灣科技大學九十六學年度碩士班招生試題

系所組別： 工業管理系碩士班甲組、乙組、丙組

科　　目： 統計學

注意事項：

1. 本試題共【4】題，配分共 100 分。請按順序標明題號作答，不必抄題。
2. Show all your calculations。

(25 %)

1. Let X and Y have the joint probability density function

$$f(x,y)=c\cdot e^{-(x+y)},\ 0\le x\le 2y<\infty$$

(1) Determine the appropriate value of c. (5 %)

(2) What is the marginal distribution of Y ? (6 %)

(3) Compute the value of E(X). (6 %)

(4) Find the probability density function of $W=X+Y$. (8 %)

(25 %)

2. Answer the following questions if a sample contains five values 1.2, 2.3, 3.9, 4.6, and 5.0.

(1) Which one of the following uniformly distributions is the possible answer to generate above sample? Explain your answer clearly. (7 %)

(a) $f(x)=\dfrac{1}{\theta},\ \theta\le x\le 2\theta$

(b) $f(x)=\dfrac{1}{2\theta},\ \theta\le x\le 3\theta$

(c) $f(x)=\dfrac{1}{3\theta},\ \theta\le x\le 4\theta$

(d) $f(x)=\dfrac{1}{4\theta},\ \theta\le x\le 5\theta$

(2) If above sample is assumed to be from the distribution

$$f(x)=\frac{2x}{\theta^2},\ 0\le x\le\theta$$

What are the maximum likelihood estimator (say $\hat{\theta}$) and estimate of the parameter θ for this sample? (6 %)

(3) Derive the density function for $\hat{\theta}$. (6 %)

(4) Determine the value of c such that $c\hat{\theta}$ is the unbiased estimator of θ. (6 %)

(30%)

3. For the simple linear model, show that $SSE=S_{yy}-\hat{\beta}S_{xy}$ (15%) and $E[SSE/(n-2)]=\sigma^2$ (15%)

where $\hat{\beta}=S_{xy}/\sum_{i=1}^{n}(x_i-\bar{x})^2$, $S_{yy}=\sum_{i=1}^{n}(y_i-\bar{y})^2$ and $S_{xy}=\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$.

(20%)

4.Suppose that $X\sim Poisson(\mu)$. Derive the most powerful test $H_0:\mu=\mu_0$ versus $H_1:\mu=\mu_1\,(\mu_1>\mu_0)$ based on an observed value of X.