國立台灣科技大學九十六學年度碩士班招生試題

系所組別： 財務金融研究所碩士班

科　　目： 統計學

總分100分

Question 1

Suppose the random vector (X, Y) is distributed with probability density,

$$f(x, y) = \begin{cases} x+y & 0 < x < 1 \text{ and } 0 < y < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find E[X]. (5 points)

(b) Find E[X + Y]. (5 points)

(c) Find E[XY]. (5 points)

Question 2

Use Chebyshev's inequality to find a lower bound on Pr(-8 < X < 16) where the random variable X has a mean $\mu = 4$ and variance $\sigma^2 = 16$. (5 points)

Question 3

In fitting a least squares line $(y = \beta_0 + \beta_1 x + \varepsilon)$ to $n = 10$ data points, the following quantities were computed:

$$SS_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{10} = 32, \quad \bar{x} = \frac{\sum x_i}{10} = 3, \quad SS_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{10} = 26,$$

$$\bar{y} = \frac{\sum y_i}{10} = 4, \text{ and } SS_{xy} = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{10} = 28.$$

(a) Find the least squares line. (5 points)

(b) Calculate *SSE*. (5 points)

(c) Calculate the estimated variance (s^2) of random error (ε). (5 points)

(d) Find a 95% confidence interval for the mean value of y when $x = 2.5$. (5 points)

(e) Find a 95% prediction interval for y when $x = 4$. (5 points)

(f) Calculate the coefficient of determination. (5 points)

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Question 4. (30 points)

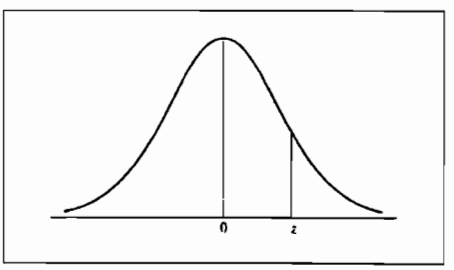
The continuous r.v. X has p.d.f. $f(x)$ where $f(x)=\frac{3}{4}\left(1+x^2\right)$, $0\le x\le 1$. If $E(X)=\mu$ and $Var(X)=\sigma^2$, find $P\left(|X-\mu|<1.64\sigma\right)$.

Question 5.

Let $Y_1<Y_2<Y_3<Y_4$ be the order statistics of a random sample of size n = 4 from a distribution with the uniform p.d.f. $f(x)=1$, $0<x<1$.

(a) Find the p.d.f. of Y_3. (10 points)

(b) Compute $P\left(\frac{1}{3}<Y_3<\frac{2}{3}\right)$. (10 points)



The entries in Table I are the probabilities that a random variable having the standard normal distribution will take on a value between 0 and z. They are given by the area of the gray region under the curve in the figure.

TABLE I NORMAL-CURVE AREAS

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Also, for z = 4.0, 5.0, and 6.0, the areas are 0.49997, 0.4999997, and 0.499999999.

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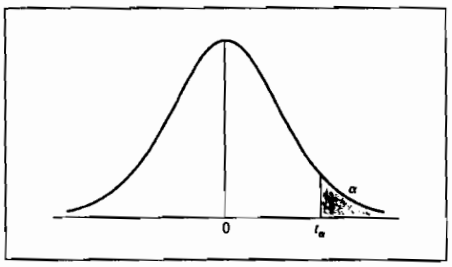

The entries in Table II are values for which the area to their right under the t distribution with given degrees of freedom (the gray area in the figure) is equal to α.

TABLE II VALUES OF t[†]

d.f.	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$	d.f.
1	6.314	12.706	31.821	63.657	1
2	2.920	4.303	6.965	9.925	2
3	2.353	3.182	4.541	5.841	3
4	2.132	2.776	3.747	4.604	4
5	2.015	2.571	3.365	4.032	5
6	1.943	2.447	3.143	3.707	6
7	1.895	2.365	2.998	3.499	7
8	1.860	2.306	2.896	3.355	8
9	1.833	2.262	2.821	3.250	9
10	1.812	2.228	2.764	3.169	10
11	1.796	2.201	2.718	3.106	11
12	1.782	2.179	2.681	3.055	12
13	1.771	2.160	2.650	3.012	13
14	1.761	2.145	2.624	2.977	14
15	1.753	2.131	2.602	2.947	15
16	1.746	2.120	2.583	2.921	16
17	1.740	2.110	2.567	2.898	17
18	1.734	2.101	2.552	2.878	18
19	1.729	2.093	2.539	2.861	19
20	1.725	2.086	2.528	2.845	20
21	1.721	2.080	2.518	2.831	21
22	1.717	2.074	2.508	2.819	22
23	1.714	2.069	2.500	2.807	23
24	1.711	2.064	2.492	2.797	24
25	1.708	2.060	2.485	2.787	25
26	1.706	2.056	2.479	2.779	26
27	1.703	2.052	2.473	2.771	27
28	1.701	2.048	2.467	2.763	28
29	1.699	2.045	2.462	2.756	29
inf.	1.645	1.960	2.326	2.576	inf.

[†] Richard A. Johnson and Dean W. Wichern, *Applied Multivariate Statistical Analysis*, © 1982, p. 582. Adapted by permission of Prentice Hall, Upper Saddle River, N.J.