國立台灣科技大學九十七學年度碩士班招生試題

系所組別： 電子工程系碩士班乙一組

科　　目： 工程數學

總分 100 分

1. **(20 Points)** Briefly answer the following questions. You will not get any credit if only the answer is given. Each problem worths 4 points.

   (a) Let $\mathbf{A}$ be an 4×4 matrix which satisfies

   $$\mathbf{a}_1 + 2\mathbf{a}_2 - 4\mathbf{a}_4 = \mathbf{0}$$

   where $\mathbf{a}_i$ denotes the i^{th} column of $\mathbf{A}$ and $\mathbf{0}$ is a 4×1 zero vector, then how many possible solutions will the system $\mathbf{Ax} = \mathbf{b}$ have? Explain.

   (b) Let $\mathbf{B}$ be another 4×4 matrix, is $\mathbf{C} = \mathbf{AB}$ singular? Briefly justify your answer.

   (c) Determine the row echelon form of $\mathbf{xy}^T$, where $\mathbf{x}$ and $\mathbf{y}$ are two nonzero (column) vectors in $\mathcal{R}^n$.

   (d) (c) continued. Determine the dimension of the null space of $\mathbf{xy}^T$.

   (e) Find the inverse of the following block matrix

   $$\begin{bmatrix} \mathbf{0} & -\mathbf{I} \\ -\mathbf{I} & \mathbf{H} \end{bmatrix}$$

   where $\mathbf{0}$ is an $n \times n$ zero matrix, $\mathbf{I}$ is an $n \times n$ identity matrix, and $\mathbf{H}$ is an $n \times n$ invertible matrix.

2. **(10 Points)** Let $\mathbf{P}_n$ denote an inner product space which consists of all polynomials of degree less than n with the inner product defined as $\langle p(x), q(x) \rangle = \int_0^1 p(x)q(x)\,dx$. Now suppose that U is the subspace of $\mathbf{P}_3$ which is given by

   $$\mathsf{U} = \{r(x) : r(0) = 0\}$$

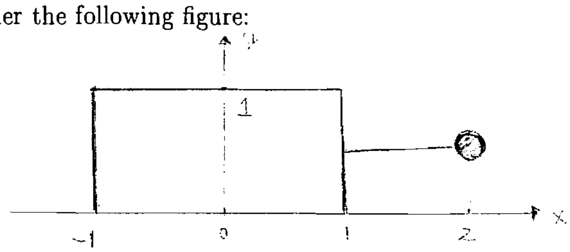
   (a) **(5 Points)** Determine an *orthogonal* basis for U.

   (b) **(5 Points)** Consider another subspace of $\mathbf{P}_3$ which is given by

   $$\mathsf{V} = \{t(x) : t(-1) = 0\}$$

   Determine dim($\mathsf{U} \cap \mathsf{V}$).

3. **(10 Points)** Consider the following figure:



   (a) **(5 Points)** If all the points in the above figure undergo the linear transformation of $\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$. Plot the resulting figure.

   (b) **(5 Points)** Is the above linear transformation one-to-one? Is the above linear transformation onto? Justify your answer.

4. **(10 Points)** As we learned in Linear Algebra, the adjoint of an $n \times n$ matrix $\mathbf{A}$ is defined as

   $$\text{adj } \mathbf{A} = \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}$$

   where A_{ij} is the cofactor of a_{ij}. Also, it is known that the adjoint satisfies the property that $\mathbf{A} \cdot \text{adj } \mathbf{A} = \det(\mathbf{A})\mathbf{I}$. Now suppose that $\mathbf{A}$ has eigenvalues $\lambda_1, \cdots, \lambda_n$, then

   (a) **(5 Points)** Determine $\text{Trace}\big(\text{adj } \mathbf{A}\big)$ in terms of the eigenvalues of $\mathbf{A}$, where Trace$(\cdot)$ denotes the summation of the diagonal elements of the matrix inside.

   (b) **(5 Points)** Determine $\det\big(\text{adj } \mathbf{A}\big)$ in terms of the eigenvalues of $\mathbf{A}$.

國立台灣科技大學九十七學年度碩士班招生試題

系所組別： 電子工程系碩士班乙一組

科　　目： 工程數學

5. The random variable X is selected at random from the unit interval; the random variable Y is then selected at random from the interval (0,X). Find the cdf of Y.

(14%)

6. Let X be the input to a communication channel and let Y be the output. The input to channel is +1 volt or -1 volt with equal probability. The output of channel is the input plus a noise voltage N that is uniformly distributed in the interval from +2 volts to -2 volts. Find $P[X = +1, Y \leqq 0]$ and the probability that Y is negative given that X is +1.

(14%)

7. A particle leaves the origin under the influence of the force of gravity and its initial velocity v forms an angle φ with the horizontal axis. The path of the particle reaches the ground at a distance

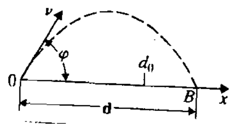
$$d = \frac{v^2}{g} \sin 2\varphi$$

from the origin (Fig 1). Assuming that φ is a random variable uniform between 0 and $\pi/2$, determine : the probability that $d \leqq d_0$. (14%)

國立台灣科技大學九十七學年度碩士班招生試題

系所組別： 電子工程系碩士班乙一組

科　　目： 工程數學



(Fig1)

8. prove $f(x|\mathbf{X} \le a) = \frac{f(x)}{\int_{-\infty}^{a} f(x)dx}$ for $x < a$

And $f(x|b < \mathbf{X} \le a) = \frac{f(x)}{F(a)-F(b)}$ for $b \le x < a$

(8%)