

## 國立台灣科技大學九十七學年度碩士班招生試題

系所組別：財務金融研究所碩士班

科目：統計學

※ 總分 100 分

1. A sociologist hypothesizes that later-born children maintain closer relationships with their families than do their older brothers and sisters. To check her hypothesis, she selects a random sample of 20 college students from each of three groups. Group 1 consists of students who are oldest children, group 2 of students who have both older and younger siblings, and group 3 of students who are youngest children. She then obtains information about the number of letters written home by each student over a period of 6 months at college. The results are as follows.

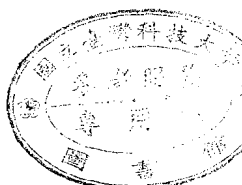
| Letter frequency          | Family Position |        |          |
|---------------------------|-----------------|--------|----------|
|                           | Oldest          | Middle | Youngest |
| At least twice per month  | 7               | 14     | 12       |
| Less than twice per month | 13              | 6      | 8        |

Calculate the appropriate statistic for this data under the hypothesis that family position has no effect on letter writing frequency and show how to justify this hypothesis. (10 points)

2. Experience has shown that while walking in a certain park, the time  $X$ , in minutes, between seeing two people smoking has a density function of the form as follows:

$$f(x) = \begin{cases} \lambda x e^{-x} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Calculate the value of  $\lambda$ . (5 points)  
 (b) Find the probability distribution function of  $X$ . (5 points)  
 (c) What is the probability that Jack, who just seen a person smoking, will see another person smoking in 2 to 5 minutes? In at least 7 minutes? (10 points)



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3. Consider the following data and models:

| obs. | $x_i$ | $y_i$ | $x_i^2$ | $y_i^2$ | $x_i y_i$ | $(x_i - \bar{x})^2$ | $(y_i - \bar{y})^2$ | $(x_i - \bar{x})(y_i - \bar{y})$ |
|------|-------|-------|---------|---------|-----------|---------------------|---------------------|----------------------------------|
| 1    | 1     | -3    | 1       | 9       | -3        | 4                   | 1.44                | -2.4                             |
| 2    | 2     | 8     | 4       | 64      | 16        | 1                   | 148.84              | -12.2                            |
| 3    | 2     | 1     | 4       | 1       | 2         | 1                   | 27.04               | -5.2                             |
| 4    | 2     | 2     | 4       | 4       | 4         | 1                   | 38.44               | -6.2                             |
| 5    | 3     | 0     | 9       | 0       | 0         | 0                   | 17.64               | 0.0                              |
| 6    | 3     | -7    | 9       | 49      | -21       | 0                   | 7.84                | 0.0                              |
| 7    | 4     | -9    | 16      | 81      | -36       | 1                   | 23.04               | -4.8                             |
| 8    | 4     | -15   | 16      | 225     | -60       | 1                   | 116.64              | -10.8                            |
| 9    | 4     | -2    | 16      | 4       | -8        | 1                   | 4.84                | 2.2                              |
| 10   | 5     | -17   | 25      | 289     | -85       | 4                   | 163.84              | -25.6                            |
| Sum  | 30    | -42   | 104     | 726     | -191      | 14                  | 549.60              | -65.0                            |

Model 1:  $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ Model 2:  $y_i = \beta_0 + \varepsilon_i$ Model 3:  $y_i = \beta_1 x_i + \varepsilon_i$ 

- (a) Calculate the least squares estimates (i.e.,  $\hat{\beta}_0$  and  $\hat{\beta}_1$ ) and corresponding standard error (i.e.,  $S_{\hat{\beta}_0}$  and  $S_{\hat{\beta}_1}$ ) for the  $\beta_0$  and  $\beta_1$  coefficients of the Model 1. (10 points)
- (b) Calculate the least squares estimates (i.e.,  $\hat{\beta}_0$ ) and corresponding standard error (i.e.,  $S_{\hat{\beta}_0}$ ) for the  $\beta_0$  coefficient of the Model 2. (5 points)
- (c) Calculate the least squares estimates (i.e.,  $\hat{\beta}_1$ ) and corresponding standard error (i.e.,  $S_{\hat{\beta}_1}$ ) for the  $\beta_1$  coefficient of the Model 3. (5 points)



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4. (30%) Supposed we took a pair of random sample  $(X_1, X_2)$  with replacement from population  $f(x)$

|        |     |     |     |     |
|--------|-----|-----|-----|-----|
| X      | 0   | 1   | 2   | 3   |
| $f(x)$ | 0.2 | 0.3 | 0.4 | 0.1 |

- (1) Construct the sampling distribution of  $\bar{X}$ .
- (2) Find  $E(\bar{X})$ .
- (3)  $\text{Var}(\bar{X})$ .
- (4) Find  $P(\bar{X} > 1)$ .
- (5)  $P(X_1 = 3, X_2 \geq 2)$ .
- (6)  $\text{Cov}(X_1, X_2)$ .

5. (20%) Two hypotheses concerning the probability density function of a random variable  $X$  are

$$H_0: f(x) = \begin{cases} \frac{1}{4}(x+1) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$H_1: f(x) = \begin{cases} \frac{1}{4}x^3 & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

- (1) Sketch the  $p.d.f.$  in each case.

The following test procedure is decided upon. A single observation of  $X$  is made and if  $X$  is less than a particular value  $k$ , where  $0 < k < 2$ , then  $H_0$  is accepted, otherwise  $H_1$  is accepted.

- (2) Find  $k$  if  $P(\text{Type I error})$  is 0.1.
- (3) With this value of  $k$ , find  $P(\text{Type II error})$ .

