

國立台灣科技大學九十八學年度碩士班招生試題

系所組別： 自動化及控制研究所碩士班甲組、乙組

科 目： 工程數學

(總分為 100 分)

1. For the differential equation

$$(5x^3 + 12x^2 + 6y^2) dx + 6xy dy = 0$$

- (a) Show that the differential equation is not exact and find its integration factor. (10%)
- (b) Solve the differential equation (10%)

2. Find the solution of the initial value problem by Laplace transform. (15%)

$$y'' + 4y' + 13y = e^{-t}, \quad y(0) = 0, \quad y'(0) = 2.$$

3. Calculate the Fourier series of the following periodic function. (15%)

$$f(x) = |\sin x|, \quad -\pi < x < \pi$$

4. Evaluate $\oint_C \left(\frac{-y}{x^2 + y^2} + 3x \right) dx + \left(\frac{x}{x^2 + y^2} - y \right) dy$ with any simple closed path C in the x-y plane that does not pass through the origin for the following cases:

- (a) The closed path C does not enclose the origin. (10%)
- (b) The closed path C does enclose the origin. (10%)

5. Let C be a circle given by $|z+1+\frac{i}{2}| = 4$ and calculate the integral $\oint_C f(z) dz$ for the following complex function. (15%)

$$f(z) = \frac{e^{iz}}{z^2 + 9}$$

6. Apply Fourier transform method to solve the following differential equation for all real t . (15%)

$$y' - ay = H(t)e^{-at}$$

where $H(t)$ is a unit step function and a is a positive constant.

$$\text{Note: } F \left\{ e^{-a|t|} \right\} (w) = \frac{2a}{a^2 + w^2}, \quad a > 0$$

