

國立台灣科技大學九十八學年度碩士班招生試題

系所組別：機械工程系碩士班乙組、丙組、丁組

科目：工程數學

總分 100 分，每題 20 分。

1. Solve the initial value problem:

$$(x^2 - 2x)y' + (x^2 - 5x + 4)y = (x^4 - 2x^3)e^{-x}; y(3) = 18e^{-3}$$

2. Solve the initial value problem by using the Laplace transform

$$y'' + 5y' + 6y = f(t); y(0) = y'(0) = 0 \quad \text{with}$$

$$f(t) = \begin{cases} -2 & \text{for } 0 \leq t < 3 \\ 0 & \text{for } t \geq 3 \end{cases}$$

3. Find the eigenvalues of matrix
- $\begin{pmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{pmatrix}$
- . Find an orthogonal matrix that

diagonalizes the matrix.

4. Solve for the integral
- $\oint_c \frac{i \sin z}{2z - \pi} dz$
- , where
- c
- is a simple closed contour enclosing

$$z_0 = \frac{\pi}{2}.$$

Hint: Cauchy's integral formula can be used.

5. Develop the following function
- $f(z)$
- into a Maclaurin series

 $(a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \dots)$ at least 4 leading non zero terms.

$$f(z) = \frac{2+z}{(1+z)^2}$$

