

國立台灣科技大學九十八學年度碩士班招生試題

系所組別：機械工程系碩士班丁組

科目：系統控制

(總分為 100 分)

題目共四題，總分 100 分

1. The system $G(s) = \frac{25}{s^2 + 14s + 25}$ is being controlled using the unity feedback structure, as shown in Fig. 1.

- (a) The first design using a P-type controller, i.e., $G_c(s) = K_p$, is proposed, verify that the closed-loop system is always stable under P-control using root locus and frequency response analyses. [5%]
- (b) To improve the steady state error, one proposes to use a PI-type controller, i.e., $G_c(s) = K_p + \frac{K_I}{s}$, and suppose that K_p has been chosen to be 3 already. Sketch the root locus of this design with K_I as the varying gain. Find the center and angles of the asymptotes, the $j\omega$ -axis crossings, the break-in and break-away point, and the angles of departure/arrival whenever applicable. If any of the above phenomena does not occur, indicate "not-applicable" as the answer. [20%]
- (c) Verify whether the system is still always stable under the varying K_I , again, use root locus and frequency response to justify your claim. [5%]

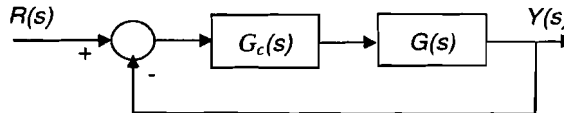
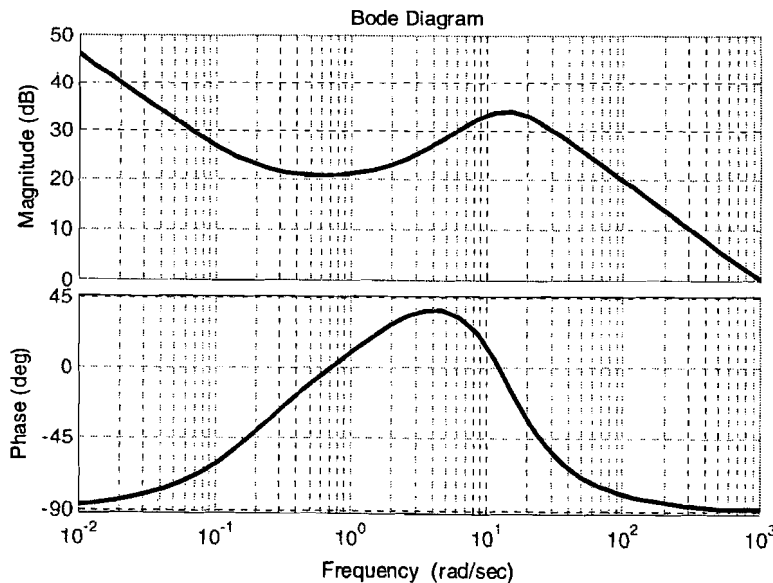


Figure 1: Unity feedback system structure

2. Based on the Bode plot shown below,
- (a) Give, from your judgment, the number of poles and zeros of the system, and their estimated locations. [5%]
- (b) Roughly sketch the corresponding Nyquist plot. [5%]
- (c) Find the system type, gain margin, and phase margin. [5%]
- (d) Suppose a sinusoidal input $u(t) = 2\cos 0.2t$ is fed to the system, describe the output signal at steady state using phasor representation. [5%]



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3. Consider a LTI system with the following transfer function $G(s)$:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10000}{s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}$$

where $U(s)$ is the Laplace transform of the input $u(t)$, and $Y(s)$ is the Laplace transform of the output $y(t)$. When the input $u(t) = n(t)$, where $n(t)$ is a unit step, and one measures the output $y(t)$ at sampling frequency 20Hz and obtained the following table:

t	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$y(t)$	0.000	0.000	0.001	0.004	0.012	0.027	0.049	0.074	0.096	0.113	0.121

t	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05
$y(t)$	0.120	0.115	0.107	0.101	0.097	0.096	0.096	0.096	0.096	0.095	0.095

When the input $u(t) = m(t)$, where $m(t)$ is a unit impulse, and again at sampling frequency 20Hz one obtains the output $y(t)$ as follows:

t	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$y(t)$	0.000	0.002	0.027	0.104	0.233	0.377	0.478	0.491	0.404	0.246	0.070

t	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05
$y(t)$	-0.069	-0.141	-0.144	-0.102	-0.049	-0.011	0.003	-0.001	-0.009	-0.008	0.005

Please answer the following questions:

- Assuming zero initial condition, when the input $u(t) = 100m(t)$, please compute or estimate the following outputs at different time: $y(0.1)$, $y(0.3)$, $y(0.6)$, and $y(0.7)$. [5%]
- When the input $u(t) = 1000m(t) + 600n(t)$, please compute or estimate the following outputs: $y(0.2)$, $y(0.6)$, and $y(0.72)$. [10%]
- Use the above to consider a different system with the following transfer function

$$\hat{G}(s) = \frac{Y(s)}{U(s)} = \frac{50000s + 20000}{s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}$$

This system has the same denominator as that of $G(s)$, but with a different numerator. When the input is a unit step, please compute or estimate its outputs at the following different time: $y(0.2)$, $y(0.5)$, $y(0.8)$, and $y(1.0)$. [10%]

4. Please answer the following questions:

- Consider a time function given by the following equation:

$$f(t) = p_1\delta(t-6) + p_2 \exp(-2.2t) \sin(3t+1.7) + p_3u(t) + p_4u(t-8) + p_5 \cos(5t-0.8)$$



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where $\delta(t)$ is a unit impulse function, $u(t)$ is a unit step function, and p_i 's are constants.

Please compute or estimate the bounds of the steady state of $f(t)$, i.e., if $f_m \leq f(t)|_{t \rightarrow \infty} \leq f_M$,

then what f_m and f_M will be? [10%]

- (b) Consider a time function given by $f(t) = \sum_{i=1}^N q_i \exp(b_i t) \cos(\omega_i t + \theta_i)$ and we know that it is generated by a LTI system $G(s)$, please answer the following questions ($G(s)$ will be different in each of the following questions):

- (1) If $N = 3$; $q_1 = 2$, $q_2 = 0.5$, $q_3 = 0.67$;

$$b_1 = -0.08, b_2 = -2.59, b_3 = 0.002;$$

$$\omega_1 = 5.92, \omega_2 = 0, \omega_3 = 7.91;$$

$$\theta_1 = 0.27, \theta_2 = 0, \text{ and } \theta_3 = 1.29,$$

what is the order of $G(s)$? how many poles in this system? What is the natural frequency of the dominant pole? What $f(t)$ will be when $t \rightarrow \infty$? [10 %]

- (2) If $N = 2$; $q_1 = 2$, $q_2 = 6.5$;

$$b_1 = -2.15, b_2 = -5.57;$$

$$\omega_1 = 0.62, \omega_2 = 1.85;$$

$$\theta_1 = 0.37, \theta_2 = 2.22,$$

what is the order of $G(s)$? how many poles in this system? What is the natural frequency of the dominant pole? What $f(t)$ will be when $t \rightarrow \infty$? [5%]

