

國立台灣科技大學九十八學年度碩士班招生試題

系所組別：化學工程系碩士班

科目：工程數學

(總分為 100 分。請依序作答，並詳列計算過程)

- The following first order ordinary equations (a) $x - xy - y' = 0$; (b) $y' - \frac{1}{x}y + \frac{2}{x} = \frac{1}{x}y^2$; (c) $xy' = \frac{y^2}{x} + y$ each one may belong to one or more of the following types: separable, linear differential equation, exact differential equation, homogeneous differential equation, Bernoulli equation or Riccati equation. Just write down what type (types) each one belongs to, no need to solve the differential equation. (15%)
- Find the general solution of the second order ordinary differential equation $x^2 y'' + xy' + 4y = 2 \ln x$ (10%)
- Find $f(t)$ and A , if $\mathcal{L}\{f(t)\} = F(s) = s \ln \frac{s-1}{s+1} + A$ where A is a constant. (15%)
- Find two linearly independent solutions of the second order ordinary differential equation $(1 - x^2)y'' - 2xy' + 6y = 0$ (10%)
- Solve $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ for $0 < x < \infty, t > 0$
with boundary condition: $y(0, t) = 0$ for $t \geq 0$
and initial conditions: $y(x, 0) = 0$ and $\frac{\partial y}{\partial t}(x, 0) = f(x)$ for $0 < x < \infty$ (18%)
- Find the Fourier series of function $f(x) = 1 - |x|$ on $-2 \leq x \leq 2$. (12%)
- Find the unique solution of the system, using the theorem that the unique solution is $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ for a nonhomogeneous system $\mathbf{AX} = \mathbf{B}$ when \mathbf{A} is nonsingular.
 $4x_1 + 6x_2 - 3x_3 = 0$ (12%)
 $2x_1 + 3x_2 - 4x_3 = 0$
 $x_1 - x_2 + 3x_3 = -7$
- Determine all values of $(7i)^{3i}$.
Hint: $\log(z) = \ln(|z|) + i \arg(z)$ for a complex number z . (8%)

