

國立台灣科技大學九十八學年度碩士班招生試題

系所組別：電機工程系碩士班丙組

科目：信號與系統

總分 100 分，請依序作答

- (1) Prove the following right-shift properties of one-sided z-transform given by the following formula, suppose that $x[n] \rightarrow X(z)$

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

- (a) (5%) $x[n-1] \rightarrow z^{-1}X(z) + x[-1]$
 (b) (5%) $x[n-2] \rightarrow z^{-2}X(z) + z^{-1}x[-1] + x[-2]$

- (2) For the discrete-time system given by the difference equation

$$y[n] + 1.5y[n-1] + 0.5y[n-2] = x[n] - x[n-1]$$

The input $x[n]$ is the unit-step function.

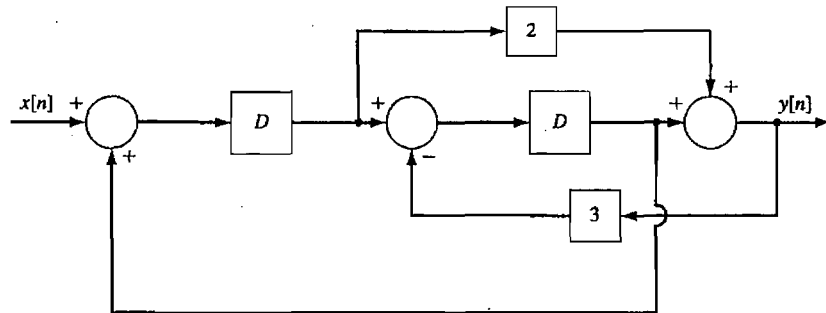
- (a) (5%) Compute $y[n]$ for $n=0, 1, 2, 3$ given that $y[-2]=1, y[-1]=2$
 (b) (5%) Find $Y(z)$ using the right-shift properties of one-sided z-transform in (1)
 (c) (5%) Find $y[n]$ by taking of inverse z-transform of $Y(z)$ and compare your answers in (a) and (c)

- (3) (10%) For the system $H(\Omega) = 1 + e^{-j\Omega}$ and the sinusoid input

$$x[n] = 2 + 2\sin\left(\frac{\pi}{2}n\right), \text{ find the output } y[n].$$

- (4) (15%) Consider the block diagram of the discrete-time system, in which the block D represents unit delay. By assuming zero initial condition, find the transfer

$$\text{function in the form of } H(z) = \frac{b_2z^2 + b_1z + b_0}{a_2z^2 + a_1z + a_0}.$$



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(5) Suppose that the continuous-time LTI system is defined by the following I/O relation: $y(t) = x(t+1) + 2x(t) + x(t-2)$

- (a) (5%) Calculate the impulse response $h(t)$
- (b) (10%) Calculate the frequency response $H(j\omega)$
- (c) (5%) Plot the magnitude of $H(j\omega)$

(6) Consider a periodic signal, $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$, namely periodic impulse train,

where the period is denoted by T_s .

- (a) (5%) Plot the time-domain signal $p(t)$ vs. t
- (b) (5%) Calculate the Fourier coefficients $\{a_k\}$ by using Fourier series integral

(7) Fourier transform and convolution

(a) (10%) Find the time-domain signal $x(t)$ of $X(j\omega)$ with inverse Fourier transform

$$X(j\omega) = \begin{cases} 1, & |\omega| \leq 100\pi \\ 0, & |\omega| > 100\pi \end{cases}$$

(b) (5%) Calculate the following time-domain signal $x(t)$ by convolution.

$$x(t) = \frac{\sin(100\pi t)}{\pi} * \frac{\sin(200\pi t)}{\pi} * \frac{\sin(300\pi t)}{\pi} * \frac{\sin(400\pi t)}{\pi}$$

(8) (5%) Shannon's sampling theorem tells us that "A continuous-time signal $x(t)$ with frequencies no high than f_{\max} can be reconstructed exactly by its samples $x[n]=x(nT_s)$, if the samples are taken at a rate $f_s=1/T_s$ that is greater than $2f_{\max}$ ". Please give me an example in the electrical engineering field which applies sampling theorem on the system design.

