（總分爲 100 分）

Problem 1

(15%)　Consider a parallel RLC resonator circuit shown in Fig. 1. It can be shown that the differential equations describing the currents i_1 and i_2 are given by

$$L\frac{di_1}{dt} + Ri_2 = E(t)$$

$$RC\frac{di_2}{dt} + i_2 - i_1 = 0$$

Solve the current $i_1(t)$ flowing through the inductor if　$L = \frac{1}{2}$ H, C = 1 F, R = 1 Ω, and　$E(t) = e^{-t}, t > 0$. The initial conditions are $i_1(0)=0$ and　$i_1'(0)=1$.
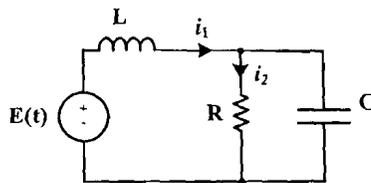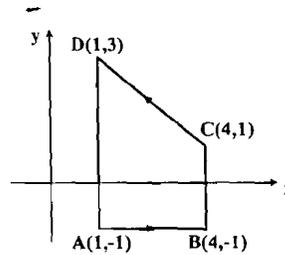

Fig. 1　　　　　Fig. 2

Problem 2

(a) (5 %) Find the curl of the following vector in Cartesian coordinate system.

$$\mathbf{F} = 2x\mathbf{a}_y$$

(b) (10 %) Using the result in (a), evaluate the line integral　$\oint_C \mathbf{F} \cdot d\mathbf{l}$, where C is a closed path ABCDA shown in Fig. 2.

(c) (5 %) Find the divergence of the vector you derived in (a).

Problem 3

(15 %) Evaluate the integral over the path　$\oint_\Gamma \frac{\cos(z)}{z^2 + 4} dz$; Γ　is a square of side length

4. The square Γ is centered at $-2j$ whose sides are parallel to the axes. ($j = \sqrt{-1}$)

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系所組別： 電機工程系碩士班己組

科　　目： 工程數學

（總分為 100 分）

Problem 4

Let matrix $\mathbf{A}$ of size 3-by-3 be represented as:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

(a). (15 %) Solve the linear equation $\mathbf{Ax} = \mathbf{b}$ by determining the unknown vector $\mathbf{x}$ with the known 3-by-1 column vector $\mathbf{b} = [0\ 9\ 3]^T$, where $(\cdot)^T$ indicates transpose operation.

(b). (10 %) Let matrix $\mathbf{A}_z = \mathbf{A} - 3\mathbf{I}$, where $\mathbf{I}$ is an identity matrix of dimension 3, i.e.

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ Determine if the vector set, composed of the three column

vectors in matrix $\mathbf{A}_z$, is linearly independent or not.

Problem 5

Let a linear time-invariant (LTI) system be described by the following difference equation:

$$y[n] = x[n] - x[n-1]$$

where $x[n]$ is the input and $y[n]$ is the output signal.

(a). (10 %) What is the frequency response of this LTI system? You can do this by first determining the impulse response followed by the discrete-time Fourier transform of it.

(b). (10 %) If for any time instant n ($n = 0, 1, 2 \ldots$) both inputs $x[n]$ and $x[n-1]$ are assumed to be identically independent distributed (*i.i.d.*) Gaussian random variables with zero mean and variance of 2. Determine the mean and variance of random variable $y[2]$.

(c). (5 %) Determine the probability of $P(y[2] < 0)$.