國立台灣科技大學九十八學年度碩士班招生試題

系所組別： 財務金融研究所碩士班

科　　目： 統計學

Statistics

共五題, 總計 100 分。依序作答。

1. (20%) Assume an investor has the utility function $W^{1/2}$, where W is wealth. The government issues a lottery ticket that pays the winner NT$300. The lottery ticket costs NT$1. The chance of winning the lottery is 1/200. The investor has NT$270 in original wealth.

   (a) What is the expected value of this lottery?

   (b) What is the investor's expected utility if he buys the lottery?

   (c) Is the investor a risk-averter, risk-neutral, or risk-lover? Use the lottery case to show it.

   (d) Will the investor buy the lottery ticket if his utility function is W^3? Use the lottery case to show it.

2. (30%) Consider a simple linear regression model $Y_i = \alpha + \beta X_i + \epsilon_i,\ i = 1, 2, \ldots, n.$ You are given the following information for independent variable X and dependent variable Y.

sample covariance $\widehat{\mathrm{Cov}}(X,Y) = 2220.48$	$\bar{Y} = \sum_{i=1}^{n} Y_i/n = 865.09$
$S_X^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2/(n-1) = 718.76$	
$S_Y^2 = \sum_{i=1}^{n}(Y_i - \bar{Y})^2/(n-1) = 7132.69$	$S_e^2 = \sum_{i=1}^{n}(Y_i - \widehat{\alpha} - \widehat{\beta}X_i)^2/(n-2) = 17.41^2$
$\bar{X} = \sum_{i=1}^{n} X_i/n = 31.18$	$n = 11$

   (a) Compute the least-squares estimate $\widehat{\beta}$ for the slope β.

   (b) Compute the least-squares estimate $\widehat{\alpha}$ for the intercept α.

   (c) Compute the t-value for $\widehat{\beta}$.

   (d) Construct a 95 percent confidence interval for β.

   (e) Test the significance of the sample correlation coefficient r.

   (f) Explain why, given the same independent variables, the confidence interval for the mean value of Y is always narrower than the corresponding confidence interval for any other value of Y.

3. (15%) Label the statements true or false.

   (a) Your goal is to hit the target at least once. If you take three independent shots at the target (instead of one shot), you triple the chances of attaining the goal (assume each shot has the same positive chance of hitting the target).

   (b) If A and B are independent events, then $\mathrm{P}(A^c|B^c)$ is $1 - \mathrm{P}(A)$, where A^c denotes the complement of A.

   (c) If A, B are independent, then they must be disjoint.

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4. (11%) A bag contains two coins: One is a normal coin, with heads and tails, while the other is a coin with heads on both sides. A coin is chosen at random from the bag and tossed 2 times. The results are all heads. What is the probability that the coin tossed is a two-headed one?

5. (24%) A quality control engineer tries to perform a hypothesis testing on the defective rate p of a production line. The engineer keeps inspecting the items until he finds the first defective. Let T denote the total number of items the engineer inspects. The engineer wants to test the hypothesis $H_0 : p \leq 10\%$ against $H_1 : p > 10\%$.

   (a) Which of the following regions is an appropriate *critical region*? Why?

      i. $\{T \geq k\}$ for some constant k.

      ii. $\{T \leq k\}$ for some constant k.

   (b) What is the probability distribution of T?

   (c) Find the maximum *type I error rate* of the test for $k = 6$.

   (d) If $p = 20\%$, what is the *power* of the test?

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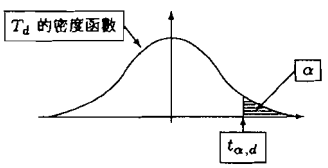

表 A.3「自由度」為 d 之 t-分配的 $100(1-\alpha)$-百分位數 $t_{\alpha,d}$

	α									
d	0.400	0.300	0.200	0.100	0.050	0.025	0.010	0.005	0.001	0.0005
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.614	1.061	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.568	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.141
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660	3.232	3.460
100	0.254	0.526	0.845	1.290	1.660	1.984	2.364	2.262	3.174	3.390
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291