

國立台灣科技大學九十九學年度碩士班招生試題

系所組別： 自動化及控制研究所碩士班甲組、乙組

科 目： 工程數學

(總分為100分)

1. The following plant model is used to describe the speed control system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -10 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

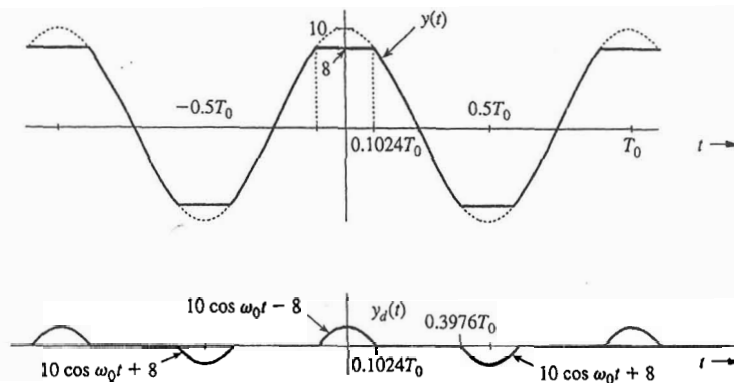
where

$x_1(t) = \omega(t)$, angular velocity of the motor shaft

$x_2(t) = i_a(t)$, armature current

Evaluate the response of this system, $y(t)$, to a unit step input $u(t) = 1$ for $t \geq 0$ under zero initial conditions. (15%)

2. The input signal to an audio amplifier is given by $y(t) = 10 \cos \omega_0 t$. However, the amplifier, being nonlinear at higher amplitude levels, clips all amplitudes beyond ± 8 as shown in the below figure.



Find the Fourier series of the distortion signal, $y_d(t)$

(20%)



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3. A thin bar of length π units is placed in boiling water (temperature 100°C). After reaching 100°C throughout, the bar is removed from the boiling water. With the lateral sides kept insulated, suddenly, at time $t = 0$, the ends are immersed in a medium with constant freezing temperature 0°C . The temperature distribution $u(x, t)$ in the bar at time t satisfies the one dimensional heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$

Find the temperature distribution $u(x, t)$ for $t > 0$. (15%)

4. Solve the following differential equations:

(a) $xyy' = x^2 + y^2, \quad x, y \neq 0$

(b) $y''' - 4y'' - 7y' + 10y = 3x - e^x$



(10%)

(10%)

5. (a) State what the Cauchy Integral Formula is. (5%)

(b) Use the Cauchy Integral Formula to find $\oint_{\Gamma} \frac{z \sin(2z - 3i)}{(z+i)^3} dz$ where Γ is the circle $|z-i|=3$. (10%)

6. About the power series expansion, answer the following:

(a) (True or False) A function $f(z)$ is analytic at a point z_0 if and only if the function is infinitely differentiable at z_0 . (2%)

(b) (True or False) If a function $f(z)$ is analytic at a point z_0 , a power series expansion of such function at that point exists, and it will be the Taylor series expansion. (2%)

(c) (True or False) The Maclaurin series is a special case of the Taylor series where the expansion point is at the origin. (2%)

(d) Explain the "ratio test" and its purpose. (3%)

(e) Use the power series expansion about the initial condition point to solve the following initial value problem, list the first five nonzero terms: $y'' + xy' = -1 + x, \quad y(2) = 1, \quad y'(2) = -4$ (6%)