

## 國立台灣科技大學九十九學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科目：工程數學

(總分為100分)

1. Find the general solution of  $y'' + 2y' + y = e^{-x} \ln x$  (20%)

2. Solve the integral equation  $f(t) = \cos(t) + e^{-2t} \int_0^t f(\alpha) e^{2\alpha} d\alpha$  (20%)

3. (i) Find the matrix inverse  $A^{-1}$  of matrix  $A = \begin{pmatrix} 1 & -3 & -4 \\ -1 & 1 & -3 \\ 0 & 1 & -3 \end{pmatrix}$

(ii) For a system of linear equations  $\mathbf{AX} = \mathbf{B}$ , in which  $\mathbf{A}$  is the  $n \times n$  matrix of

coefficients,  $\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{pmatrix}$  .. Discuss the conditions for the system

to have (a) no solution; (b) one solution; (c) more than one solution (20%)

4. Evaluate the integral  $\oint_C \bar{z} dz$  where  $C$  is the circle  $r = a$  ( $a > 0$ ). (20%)

5. Solve the boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < 1, t > 0$$

$$u(0, t) = u(1, t) = 0 \quad \text{for } t \geq 0 \quad (20\%)$$

$$u(x, 0) = 1 \quad \text{for } 0 \leq x \leq 1$$

