

## 國立台灣科技大學九十九學年度碩士班招生試題

系所組別：電機工程系碩士班丙組

科目：信號與系統

(總分為100分)

總分 100 分

1. We want to design a discrete-time LTI (Linear Time Invariant) system  $H(e^{j\omega})$  that has the property if the input is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] - \frac{1}{4} \left(\frac{1}{3}\right)^{n-1} u[n-1]$$

then the output is

$$y[n] = \left(\frac{1}{2}\right)^n u[n]$$

- (1) (5%) Find the corresponding Discrete Time Fourier Transform  $H(e^{j\omega})$ .  
 (2) (5%) Find a differential equation relating to  $x[n]$  and  $y[n]$  that characterizes this system.  
 (3) (5%) Find the impulse response  $h[n]$  of this system.
2. Given the following Discrete Time Fourier Transform (DTFT)  $X(e^{j\omega})$ , determine the associated discrete-time signal:

$$X(e^{j\omega}) = \begin{cases} 0, & 0 \leq |\omega| \leq \pi/3 \\ 1, & \pi/3 < |\omega| \leq 2\pi/3 \\ 0, & 2\pi/3 < |\omega| \leq \pi \end{cases}$$

- (1) (5%) Plot the magnitude of frequency response of  $X(e^{j\omega})$ .  
 (2) (5%) Take  $X(e^{j\omega})$  as a convolution in frequency of an ideal low pass filter  $L(e^{j\omega})$  with  $S(e^{j\omega})$ . Draw  $L(e^{j\omega})$  and  $S(e^{j\omega})$ .  
 (3) (5%) Find the inverse DTFT  $x[n]$  of the given  $X(e^{j\omega})$ .
3. Let  $X(e^{j\omega})$  be the DTFT of a symmetric signal  $x[n]$  which is defined as follows:

$$x[n] = \begin{cases} |n| - 3 & \text{when } -3 \leq n \leq 3 \\ 0 & \text{else} \end{cases}$$

- (1) (5%) Plot  $x[n]$ .  
 (2) (5%) Find  $X(e^{j0})$  without explicitly calculating  $X(e^{j\omega})$  first.  
 (3) (5%) Find  $\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$  without explicitly calculating  $X(e^{j\omega})$  first.  
 (4) (5%) Find  $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$  without explicitly calculating  $X(e^{j\omega})$  first.



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4. Let  $x(t) = \sin^3(\pi t)$ (a) (5%) Please represent  $x(t)$  as Fourier series

$$x(t) = \sum_k a_k e^{j\omega_k t}$$

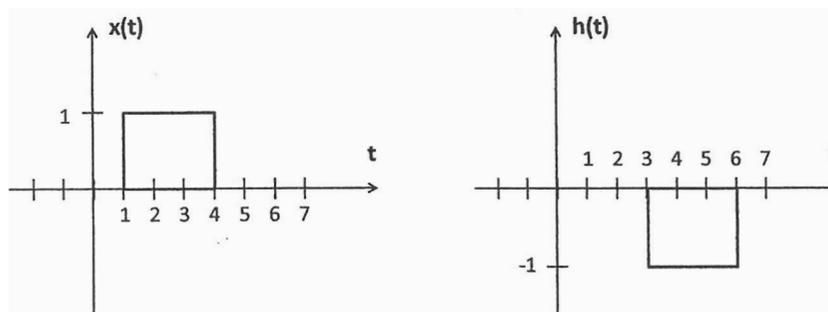
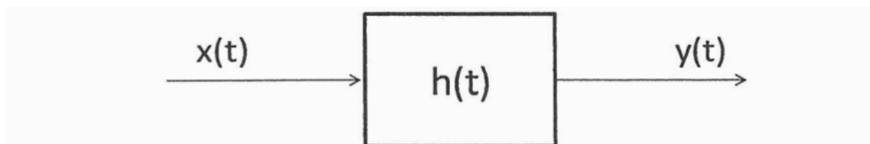
(b) (10%) Plot the spectrum of  $x(t)$ , make separate plots for magnitude and phase.

5.

Consider a LTI system shown below,

(a) (5%) Plot the output  $y(t)$  with the given input signal  $x(t)$  and the system impulse response  $h(t)$ 

(b) (10%) Calculate the frequency response of this system.



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(a) (10%) Find the Fourier transform of  $x(t) = \delta(t - 4)$ 

(b) (10%) Prove the frequency shifting property of Fourier transform

$$x(t)e^{j\omega_0 t} \stackrel{\mathcal{F}}{\Leftrightarrow} X(j(\omega - \omega_0))$$

