

國立台灣科技大學九十九學年度碩士班招生試題

系所組別：資訊工程系碩士班

科目：計算機數學

(總分為100分)

1. [15%] Consider the bases $B = \{(1, 2, 4), (-1, 2, 0), (2, 4, 0)\}$ and $B' = \{(0, 2, 1), (-2, 1, 0), (1, 1, 1)\}$ of \mathbb{R}^3 . Find the transition matrix from B to B' .

2. [10%] Find an orthonormal basis of the subspace of \mathbb{R}^3 spanned by the vectors $(1, 1, -1)$ and $(1, 0, -1)$.

3. [10%]
 - (a) (5%) True or False: If A is diagonalizable, then the rank of A equals the number of nonzero eigenvalues of A .
 - (b) (5%) Prove your answer of (a).

4. [15%]
 - (a) (10%) P_2 is a set including all polynomials of degree less than 2. Let $A \subseteq P_2$ be the vector space spanned by polynomials $x^2 + 2x + 1$, $x + 2$, $x^2 - x - 1$ and let $B \subseteq P_2$ be the vector space spanned by 1 , x , x^2 . Describe the difference set $A - B \equiv \{f(x) : f(x) \in A, f(x) \notin B\}$.
 Note: The space spanned by polynomials includes all polynomials that can be written as a linear combination of those polynomials.
 - (b) (5%) Let $f_1(x) \equiv x^2 + 2x + 1$. Consider the "multiplication" defined by the logical AND, and "addition" defined by the logical OR. That is, we define $x^2 = x \cdot x \equiv x \wedge x$ (x AND x), $2x = x + x \equiv x \vee x$ (x OR x), and $x + c \equiv x \vee c$. Evaluate $f_1(\text{FALSE}=0)$ and $f_1(\text{TRUE}=1)$.

5. [15%] Let $A = \begin{bmatrix} 1 & -1 \\ 1 & -3/2 \end{bmatrix}$.
 - (a) (7%) Compute eigenvalues and the corresponding eigenvectors of A .
 - (b) (8%) Compute $\lim_{n \rightarrow \infty} A^{2n}$.



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6. [6%] How many ways are there to pack eight identical DVDs into five indistinguishable boxes so that each box contains at least one DVD? Explain why the answer you have.
7. [6%] Determine whether the function $f(x) = (x+1)/(x+2)$ is a bijection from \mathbb{R} to \mathbb{R} . Explain why the answer you have.
8. [12%] Suppose that the function f satisfies the recurrence relation
 $f(n) = 2f(\sqrt{n}) + \log_2 n$ whenever n is a perfect square greater than 1 and $f(2) = 1$.
 (a) (4%) Find $f(16)$.
 (b) (8%) Find a big- O estimate for $f(n)$.
9. [11%]
 (a) (4%) Find the state table for the nondeterministic finite-state automaton with the state diagram as shown in Figure 1.
 (b) (7%) Find a deterministic finite-state automaton that recognizes the same language as the nondeterministic finite-state automaton in (a).

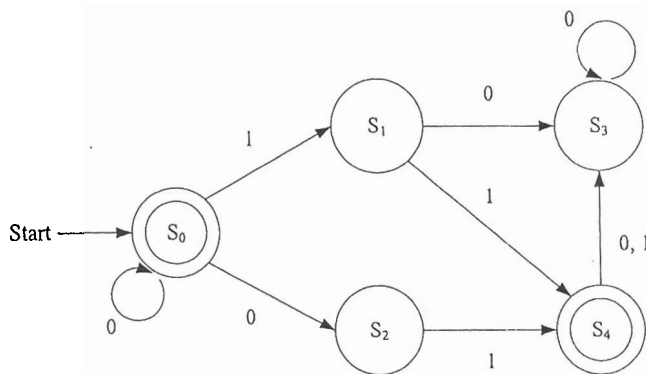


Figure 1

