國立台灣科技大學九十九學年度碩士班招生試題

系所組別： 工業管理系碩士班甲組

科　　目： 作業研究

（總分為100分）

1. There are three boxes, one each of gold, silver, and lead. A diamond is hidden in one of them. You are asked to identify the box containing the diamond. If your choice is correct, you may claim the diamond as your prize. To help you choose intelligently, inscriptions are put on the boxes as in the following figure.

The diamond is in this box	The diamond is **NOT** in this box	The diamond is **NOT** in the gold box

Gold Box　　　　　　　　Silver Box　　　　　　　　Lead Box

At most one of the three inscriptions is true. Construct a set of linear constraints, including binary variables if necessary, to identify the box containing the diamond. (10%)

2. A company produces a product at two plants, 1 and 2. The unit production cost and production capacity during each period are given in the following table.

	Unit Production Cost ($)	Capacity
Plant 1 (Period 1)	35	8
Plant 1 (Period 2)	45	5
Plant 2 (Period 1)	28	11
Plant 2 (Period 2)	40	10

The product is instantaneously shipped to the company's only customer according to the unit shipping costs given in the following table.

	Period 1	Period 2
Plant 1 to Customer	$50	$60
Plant 2 to Customer	$45	$55

If a unit is produced and shipped during period 1, it can still be used to meet a period 2 demand, but a holding cost of $15 per unit in inventory is assessed. At the end of period 1, at most five units may be held in inventory. Demands are as follows: period 1, 12; period 2, 15.

(a) Formulate a minimum cost flow problem that can be used to minimize the cost of meeting all demands on time. Draw the network diagram and determine the net flow at each node and the capacity and cost of each arc. (10%)

(b) Solve this problem by the network simplex method. (10%)

3. Consider the following linear program.

(Primal)

Maximize　　$z = 2x_1 + 3x_2 + x_3$

Subject to

$$2x_1 + x_2 + x_3 \leq 5$$
$$x_1 + 2x_2 + 3x_3 \leq 9$$
$$x_1 \leq 0, \; x_2 \leq 0$$

(a) Formulate the dual problem. (10%)

(b) Solve the dual problem by graphical method. (5%)

(c) Solve the primal problem by complementary slackness property. (5%)

4. Trials are performed in sequence. If the last two trials were successes, then the next trial is a success with probability 0.8; otherwise the next trial is a success with probability 0.5. In the long run, what proportion of trials are successes? (20%)

5. Customers arrive at a two-server system according to a Poisson process with a rate of two per hour. Arrivals finding server 1 free begin service with that server. Arrivals finding server 1 busy and server 2 free begin service with server 2. Arrivals finding both servers busy are lost. When a customer is served by server 1, she/he then either enters service with server 2 if 2 is free or departs the system if 2 is busy. A customer completing service at server 2 departs the system. The service times at server 1 and server 2 are exponential random variables with respective rates of four and six per hour.

(a) What fraction of customers does not enter the system? (10%)

(b) What is the average amount of time that an entering customer spends in the system? (10%)

(c) What fraction of entering customers receives service from server 1? (10%)